# TEACHING STATISTICS WITH A CRITICAL PEDAGOGY

A Dissertation
by
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**Abstract**

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After President Obama shifted the country's focus from K-12 towards higher education, post-secondary schools found themselves under significant public, financial, and political pressure. To close the achievement gap and meet new standards of accountability, higher education institutions began looking for methods to increase student access and success. With a growing emphasis on degree completion and quantitative literacy, some researchers began to explore the use of critical pedagogies to better serve a more diverse population (Ukpokodu, 2011). This quantitative study measured the effectiveness of implementing a critical statistics pedagogy in an undergraduate introductory statistics classroom and its impact on course success, persistence, and mathematical empowerment. Data collected from four classes at a community college found the use of a critical pedagogy had a positive impact on students' overall achievement, increased their awareness of social justice issues, and aided in the development of their critical voice.

# Acknowledgments

*If you want to go fast, go alone; but if you want to go far, go together.* It is through the endless support and guidance of my committee and family members that resulted in the completion of this dissertation. My dissertation chairperson Dr. Tracy Goodson-Espy patiently answered questions via emails and phone calls about statistical analysis to paperwork procedures and every imaginable topic in between. Dr. Lisa Poling and Dr. Tracie Salinas shared their invaluable feedback and insights on critical pedagogies and mathematics education ensuring my students received the highest quality of instruction. The knowledge the committee shared with me will forever impact my practice.

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# Chapter One: Introduction

This chapter begins with a summary statement of the problem, including relevant recent and historical events, and introduces the research questions. Next, an overview of the methodology implemented in the study and the significance of the study is presented. Necessary definitions are then established. Last, a brief outline of the organization of the study is provided, including limitations.

## Statement of the Problem

Mathematics can act as the gatekeeper from or the gateway to future success, both academically and personally (Gonzalez, 2009; Lesser & Blake, 2007). The National Council of Teachers of Mathematics (NCTM) asserted this sentiment in 2000 in the *Principles and Standards for School Mathematics* (PSSM) stating "…. mathematical competence opens doors to productive futures. A lack of mathematical competence keeps those doors closed" (p. 1). Unlike its predecessors, the PSSM emphasized the need for active learning, dialogue, and the inclusion of all stakeholders.

Shortly after the PSSM was published, the *No Child Left Behind* (NCLB) act was signed into law requiring K-12 schools to track the progress and growth of four specific subgroups: major ethnic/racial groups, economically disadvantaged students, limited English proficient students, and students with disabilities. NCLB's subgroups primarily targeted children of color, as they are more likely to be identified as special needs or at-risk students. A school was labeled as a "failing school" if the staff was not able to close the achievement gap between middle to high-income white students who were outperforming the identified subgroups on standardized tests. In February 2009 President Obama shifted the country's focus on education towards higher education, challenging the nation to have the highest proportion of college graduates in the world by 2020.

As the nation was challenged to increase the number of college graduates, higher education institutions found themselves under significant political pressure. The Lumina Foundation's Achieve the Dream (AtD) project quickly became one of the most well-known movements. AtD focused on increasing success for community college students enrolled in developmental education courses with success measured by student progression through developmental education and gatekeeper courses, persistence from term to term, and the completion of certificates and degrees (Bragg & Durham, 2012).

Building on the AtD framework, in 2011 the Gates Foundation launched a five-year initiative, Completion by Design (CbD), specifically aimed at raising community college completion rates. Seventeen institutions across three states received a total of $475 million in funding "to collaborate on the design and implementation of a model pathway to completion" (Pennington & Milliron, 2010, p. 3). Institutions associated with CbD were engaged in a systematic process of inquiry and design, aimed at systemic changes in policies, programs, and practices that strengthen pathways to completion for most students on their campuses (Baldwin, 2017). Each of the three states receiving funding had a designated 'policy lead' who oversaw the institutions statewide.

However, Complete College America (CCA) was often considered the legal springboard for the completion agenda movement (Kelly & Schneider, 2012; Walters, 2012). The CCA, a national non-profit foundation established in 2009, aimed "to significantly increase the number of Americans with quality career certificates or college degrees and to close attainment gaps for traditionally underrepresented populations" (Baldwin, 2017, p. 5). It looked to achieve this goal through influencing state and national education policies. The CCA was successful at creating a set of performance indicators to measure the success of students enrolled in colleges and universities. By November 2013, thirty-three governors committed to the CCA's higher education agenda of performance funding based upon overall completion and success rates (Rutherford & Rabovsky, 2014).

In North Carolina, performance funding for community colleges was established in 1999 with General Statute (GS) 115D-31.3 but was quickly abandoned due to budget restrictions (Dougherty & Reddy, 2013). However, with the birth of the 'completion movement' in 2009, the act was updated and reenacted in both 2012 and 2016. General Statute 115D-31.3 currently outlines seven performance indicators for all North Carolina Community Colleges. The indicators are: 1) the success of students in credit-bearing English courses, 2) performance after transfer to a four-year college, 3) the success of students in credit-bearing math courses, 4) curriculum student retention and completion, 5) first-year curriculum student progress, 6) basic skills student progress, and 7) licensure and certification passing rates ("Community College", 2016). For each measure, allocation of funds is based upon the college's performance compared to the baseline of two standard deviations below the statewide mean with a goal of being one standard deviation above the mean.

> If a college does not meet the baseline, it receives no performance-based funding. If a college exceeds the baseline but does not meet the goal, it receives a proportionate amount of eligible performance-based funding. If a college meets the goal, it receives 100% of eligible performance-based funding. If a college exceeds the goal, it receives a proportionate amount above 100% of eligible performance-based funding. For each measure, allot any remaining funds through an impact component based on the number of students meeting the measure's goal at the college relative to the number of students meeting the goal systemwide. (North Carolina General Assembly, 2016, p. 8)

In addition to the performance-based funding statue, North Carolina also established tiered funding, which represents a much larger portion of the funding formula (83%). Tiered funding allots dollars per full-time equivalent (FTE) based upon course designations and is a means to support courses that may require specialized equipment or resources. Each step “down” the tier ladder results in a 15% loss in funding. Tier 1A classes consist of healthcare and technical education curriculum courses in ‘immediate need’ areas and are awarded $4,270 per FTE. Tier 1B classes, curriculum courses in other areas of healthcare, technical education, lab-based science classes and college-level math courses, receive $3,777 per FTE. All other curriculum courses and all basic skill courses are Tier 2 and earn $3,284 per FTE. Last, Tier 3, are all other occupational extension classes which earn $2,792 per FTE.

The renewed emphasis on persistence and completion, and ties to funding models, enhanced the importance of ensuring student success in mathematics courses, forcing institutions to reevaluate their mathematics curriculums. While the PSSM targeted secondary schools, it had a ripple effect through curriculums at all levels. Much more than a mathematical road map, it exemplified a commitment to mathematical literacy and the ability to synthesize information. Until the PSSM, many mathematics curriculums only focused on problem solving and only as a means of acquiring content knowledge. However, the NCTM (2000) views problem solving as an integral piece, not an isolated one. They believed students should “be encouraged to reflect on their thinking during the problem-solving process so that they can apply and adapt the strategies they develop to other problems and in other contexts” (NCTM, 2000, p. 4). Additionally, it emphasized the importance of communicating mathematically (NCTM, 2000). Beyond improving their communication skills, students also are more likely to make connections, creating a deeper understanding of the subject matter (NCTM, 2000).

“Math has been, I think, the single biggest obstacle to retention and completion,” stated William E. Kirwan, former chancellor of the University System of Maryland (Logue, Watanabe-

Rose, & Douglas, 2016, para. 2). As the highest-level mathematics course taken in high school is the best predictor of success in a college-level mathematics course, it is not surprising a math barrier exists in higher education (Adelman, 2006; Gupta, Harris, Carrier, & Caron, 2006). To better meet the needs of a diverse student body, close the achievement gap, and meet new standards of accountability, some researchers began to explore the use of critical pedagogies, such as critical mathematics, (Ukpokodu, 2011). Critical pedagogies "are specifically concerned with the influences of educational knowledge, and of cultural formations generally, that perpetuate or legitimate an unjust status quo; fostering a critical capacity in citizens is a way of enabling them to resist such power effects" (Burbules & Berk, 1999, p. 46). Use of a mathematical critical pedagogy may be a solution to overcoming the math barrier that exists in higher education. By creating an environment where students have a "personal engagement with mathematics" they may develop an appreciation and awareness of the value of mathematics and how it can be used in their lives (Ernest, 2002, p. 12). Updating that strategy for use in a statistics course would be particularly beneficial to community colleges who have seen the number of students enrolled in statistics courses triple since 1995 (Blair, Kirkman, & Maxwell, 2015). Implementing a critical statistics pedagogy may help students complete their degree, prepare them for the workplace, and aide them in becoming more informed citizens, "as the study of statistics provides students with tools and ideas to use in order to react intelligently to quantitative information in the world" (Ben-Zvi & Garfield, 2008, p. 355).

**Research Questions**

The purpose of this study was to examine how implementing a critical pedagogy in an undergraduate statistics classroom impacts student success in the course and the student's overall belief that the material is relevant and has a powerful impact in their personal lives, hereby referred to as mathematical empowerment. Success was defined as completing the course with a C or better. Based upon the work of Gutstein (2006), Lesser (2007), and Ernest (2002), mathematical empowerment (MP) is defined as the student's ability to confidently use and apply the language, skills, and practices of mathematics, pure or applied; it represents their personal sense of power over the creation and validation of mathematical knowledge (Ernest, 2002). While MP can focus on a variety of relevant experiences, for this study MP will focus on student's ability to 'read and write the world' using statistics to make meaningful connections between injustices and oppressive structures and acts to the curriculum (Gutstein, 2006). The use of statistical methods to explore real-life scenarios could stimulate a sense of social justice possibly influencing student empowerment (Lesser & Blake, 2007).

The following research questions guided the study:

1. Does implementing a critical pedagogy increase student success? Student success is defined as completing the course with a C or higher.
2. Does use of a critical pedagogy increase the student's sense of mathematical empowerment? MP was measured using a subset of questions from the Constructivist Learning Environment Survey.

**Methodology**

During the Fall 2017 and Spring 2018 semesters four MAT 152 Statistical Methods I courses at Mitchell Community College were part of a pre-post nonequivalent groups quasi-experimental study. Students enrolled in those sections participated in the study. Students were expected to complete coursework and assessments typically assigned in a college course. The required coursework included homework, surveys, tests, labs, a pretest, posttest, and a final.

Students for all MAT 152 courses must meet the same prerequisite requirements. To be eligible to take MAT 152 students must have completed a series of developmental courses, earned a 500 on the quantitative section of the SAT or a 22 on the ACT, completed four years of mathematics in high school with at least one class past Algebra II and graduated within the last five years, or earned sufficient scores on the North Carolina Diagnostic Assessment and Placement exam (Spell, 2016). There are currently no co-requisite courses required. Class sizes for MAT 152 range between 15 and 25 students and are normally capped between 25 and 30 students depending on the room location and size. Twenty-one students registered for and completed the Fall 2017 Mooresville section, and 14 students registered for and completed the Fall 2017 Statesville section. Twenty-four students completed the Spring 2018 Mooresville section and 21 students completed the Spring 2018 Statesville section. The author of this paper was the instructor for each section participating in the study. Additionally, the author did not teach any additional sections of MAT 152 that were not included in the study in either the Fall 2017 or Spring 2018 semesters.

The control and treatment groups differed in the types of PowerPoint examples used and the themes of the instructor-created labs. The Fall 2017 semester courses served as the control groups and were instructed using the existing instructional methods and resources, both prepackaged and instructor-created. While they were real-world scenarios, the course text emphasized contrived examples such as gender selection of children or the number of chocolate chips in chocolate chip cookies. While other themes are also found in the textbook, homework, and PowerPoints, these two themes are noticeably more prominent. For the treatment groups in Spring 2018, PowerPoints were updated to focus on issues of race, class, gender, and/or sexual orientation (but not gender selection of offspring). Instructor-created labs in the control group have varying themes, but often included casino games and fictional examples. For the treatment group, all labs were updated to share the same focus as the treatment PowerPoints, race, class, gender, and/or sexual orientation, implementing only real-world data from accessible databases. The number of labs for each course per chapter remained the same. Students did not select their own areas of interest to study.

**Significance of the Issue**

The more college-level mathematics credits students earn, the more likely they are to complete a bachelor's degree (Adelman, 2006). Implementing a critical pedagogy in a community college statistics classroom may offer a multi-tiered approach to improving mathematics education and completion. First, numerous studies have shown the use of a critical pedagogy to be an effective way to raise student engagement (Gutstein, 2006; Lesser, 2007; Rouncefield, 1995). Kuh, Cruce, Shoup, Kinzie, and Gonyea (2008) defines student engagement as "both the time and energy students invest in educationally purposeful activities and the effort institutions devote to effective educational practices" (p. 542). Found to be a predictor of GPA, persistence, and completion, student engagement is regarded as an important factor in determining academic success (Alvarez-Bell, Wirtz, & Bian, 2017; Kuh et al., 2008; Zepke, 2015). As students with higher levels of engagement experience greater success, these increases not only aid students in their journey to degree completion but also offers the institution a way to meet the accountability measures outlined in GSD § 115D-31 (Alvarez-Bell et al., 2017).

Second, this approach helps to raise awareness of social justice issues and may expose biases and assumptions, empowering students to create change. Use of a critical pedagogy specifically aims to help students understand how mathematics permeates everyday life and that learning mathematics cannot be removed from understanding and the ability to influence outcomes in the real world (Ernest, 2002; Lesser & Blake, 2007). As education plays a pivotal role in developing a society's ethical, social, and cultural norms "teachers must be ethically and legally responsible to help students engage in a struggle for a more just and humane world" (Ukpokodu, 2007, p. 11).

Lastly, mathematical competency acts as a 'critical filter' in higher education (Ernest, 2002). The math barrier allows mathematics courses to exist as gatekeepers to degree completion and future career opportunities. Critical pedagogies offer a method that often results in more equitable outcomes in a mathematics classroom, with increases particularly noticeable in previously lower performing students (Wright, 2017). As mathematics qualifications are often an "admission ticket" to high-paying jobs, then a critical pedagogy may become the gateway in moving towards a more balanced economic and social structure (Lesser & Blake, 2007).

**Definitions**

Several key terms will be referred to throughout the study and in the review of the literature. Following are the terms and their definitions.

**Pedagogy.** Methods or practice of teaching; methods of instruction.

**Critical Pedagogy**. A teaching pedagogy that focuses on addressing hegemonic practices that results in the marginalization of specific groups of people (Leonard, Brooks, Barnes-Johnson, & Berry III, 2010).

**Teaching Statistics for Social Justice (TSfSJ).** A critical pedagogy implemented in a statistics curriculum exploring social justice issues. The aim of TSfSJ is to incorporate and facilitate an awareness of social justice and prepare students to be critically reflective about the role statistics plays in society (Lesser, 2007).

**Social Justice**. Promotion of the common good; equal distribution of wealth, opportunities, and privileges within a society (Oxford University Press, 2019).

**Completion.** Completing a course or a degree path; earning a final course grade or credential.

**Success.** Completing the course with a C or higher.

**Mathematical Empowerment/Power.** The ability to identify the need for and the ability to apply mathematics to real world situations and problems.

**Constructivism.** A theory learning that claims the creation of meaning is an active, emergent process developed through interactions and experiences with one's social environment (Amineh & Asl, 2015; Pierce & Hernandez, 2015).

**Dialogue.** Conversation between two or more students; collaboration.

**Discourse.** Dialogue that requires students to evaluate and interpret the perspectives, ideas, and arguments of others as well as construct valid arguments of their own; meaningful social interaction (Bennett, 2014).

**Full-Time Equivalent (FTE).** The total number of hours taken by all students at the institution divided by what is determined to be the equivalent of full-time. In North Carolina, a full-time curriculum student represents 512 hours. Chapter G, subchapter 100 of Title 1 defines one credit hour as the equivalent of 16 hours of class work (lecture), 32 hours of experimental work (lab), 48 hours of instructor-directed lab work or clinical practice, or 160 hours of work experience (i.e. – internship) (North Carolina Community College System, 2017).

**Pathway.** The sequence of courses a student must complete to earn a credential.

**Curriculum.** The content that is taught in a course; the prescribed collection of topics, skills, and subjects taught in a course or pathway.

**Developmental Education.** Courses or modules a student is required to take before taking credit-bearing courses in a given discipline.

**Gateway Course.** Course the student is required to complete before taking higher level courses in a given discipline; a prerequisite to another course.

**Overall Course Grade.** Letter grade student earned in the course; the weighted mean of all the student's graded assignments and assigned a letter grade on a 10-point scale.

**Final Exam Grade.** Grade student earned on the final exam.

**Pretest.** Test taken before curriculum is started; must contain the same test questions as on the Posttest.

**Posttest**. Test taken after all curriculum is completed; must contain the same test questions as on the Pretest.

## Organization of the Study

The purpose of the study was to examine the effect of the use of a critical pedagogy on course success and mathematical empowerment (also referred to as mathematical power) in an undergraduate level statistics course. Chapter Two presents a thorough review of the literature of critical pedagogies and constructivism. This will include a review of literature surrounding past research studies and the theoretical and conceptual framework used in this study. In Chapter Three, the methodology of the study is defined. This includes the research questions, design, and rationale for the study. The rationale specifically explores the role of the researcher, ethical issues and limitations, data sources and collection, and a description of the participants. Chapter Four is dedicated to presenting the results and descriptive analysis of the study. Conclusions and inferences based upon these results are presented in the final chapter, Chapter Five, where implications and recommendations for future research will also be included.

## Limitations

As this study utilized preconstructed groups created by self-selection, lack of randomization was a limitation of this study. The study could not account for participants' prior knowledge, prior course completions, and personal biases. Additionally, the time and length of the treatment and control groups were not negotiable or able to be changed due to state regulations. While the time frame for the sections were the same for the control and the treatment, only the participants who were available on those days and times were able to register for those sections, further limiting the reach of the study.

# Chapter Two: Literature Review

This chapter will have four parts. First, it will begin by reviewing the literature related to critical pedagogy and social constructivism, which will serve as the conceptual framework. Next, it will explore the impact of using these frameworks in a mathematics and statistics classroom. Third, it will explicitly look at community colleges and the potential impact of the using a critical pedagogy in a statistics classroom, as the study was implemented in a community college introductory statistics classroom. Last, the conceptual framework for the study is introduced and developed.

## Classic Literature

In a 2013 interview conducted by J.M.B Tristan of Global Education Magazine, Giroux defined critical pedagogy as a movement that "draws attention to questions concerning who has control over the conditions for the production of knowledge, values, and skills, and it illuminates how knowledge, identities, and authority are constructed within particular sets of social relations" (Tristan, 2013, p. 21). The term critical pedagogy (CP) was first coined in 1983 by Giroux in *Theory and Resistance in Education: A Pedagogy for the Opposition,* but it was largely founded upon the work of Paulo Freire (Kaya & Kaya, 2017; Tutak, Bondy, & Adams, 2011). Giroux's five pedagogical assumptions and practices needed to develop a CP are directly correlated with Frere's problem-posing education theory in which people develop an ability to critically examine the way they exist with and in the world through dialogue (Freire, 1970/2005).

Freire (1970/2005) asserted that learning required the learner to be active and that knowledge is created from a shared process of inquiry, interpretation, and creation. This is also Giroux's first assumption. Classrooms must be structured in a manner that will allow students the opportunity to challenge and produce knowledge (Giroux, 1983). Second, Giroux charges that students must be taught to think critically and to challenge their own modes of understanding. Freire also called for educators to engage students in “thinking which perceives reality as process, as transformation, rather than as a static entity—thinking which does not separate itself from action, but constantly immerses itself in temporality without fear of the risks involved” (Freire, 1970/2005, p. 92). Third, students must develop a sense of agency and begin to speak with their own voice (Giroux, 1983). Freire referred to this as developing a ‘critical consciousness’ so students can begin to understand their lives in new ways and see themselves as transformers of those lives (Freire, 1970/2005; Tutak et al., 2011).

The fourth and fifth assumptions presented by Giroux highlight the importance of identifying and understanding what he later refers to as “silent structures”. Students must learn the source of their beliefs and what structural and ideological factors influence them. In doing so, they can challenge the taken-for-granted ways of their world and create their own history. Freire couples these ideas into praxis which he asserted was, “the action and reflection of men and women upon their world in order to transform it” (Freire, 1970/2005, p. 79). Critical pedagogy engages students as critical thinkers, active learners, and challenges them to envision alternative possibilities (Nagada, Gurin, & Lopez, 2003). It achieves its goals through carefully constructed learning experiences that allow students to examine, apply, and reflect upon the content.

Critical pedagogy is complemented by social constructivism and offers a counter-narrative to how traditional schooling operates (Gordon, 2009). Considered the founding father of social constructivism, Vygotsky's (1934/1986) research and theories emphasized the importance of social interaction, specifically through collaboration with peers and facilitators who possess more or varied experiences. In traditional classrooms, learning is framed by a curriculum that is then taught by content experts to students. The curriculum, with detailed student learning outcomes, is distributed to schools with the expectation that students attain mastery. Such an instructional approach places the teacher as the sole, expert provider of information in the classroom; students are passive recipients of knowledge that is being "banked" into them by the teacher, where students are viewed as receptacles filled passively by teachers (Lesser & Blake, 2007). Freire's (1970/2005) problem-posing education offered a new learner-centered model of learning based on dialogue.

Dialogue in its simplest form is merely conversation, a discussion with multiple voices. Freire's dialogic education calls for students and teachers to collaboratively create learning experiences to create discourse. Discourse requires students to evaluate and interpret the perspectives, ideas, and arguments of others and construct valid arguments of their own; when students engage in meaningful social interactions, they form a deeper understanding (Bennett, 2014). According to Dewey (1938), it is only through meaningful interactions that learning can take place as learning is the by-product of those interactions. One cannot be given knowledge; one must experience and communicate with and about it.

Social constructivism also challenges the idea of knowledge creation. There are not 'objective facts'; knowledge is developed by our world experience within a socio-cultural context (Talja, Tuominen, & Savolainen, 2004; von Glaserfeld, 1984). The accepted knowledge, and the added value that is created by negotiated understanding, impacts the activities and actions of individuals. Dewey (1897/2013) supports this position stating, "Education is a regulation of the process of coming to share in the social consciousness; and that the adjustment of individual activity on the basis of the social consciousness is the only sure method of social reconstruction" (p. 39). Most importantly, social constructivism states that knowledge is shaped by social, cultural, and language-based interactions. It is through the social network that helps a culture “develop its language and the belief systems and promotes the process of enculturation” (Brown, Collins, & Duguid, 1989, p. 23). As the meanings of labels and classification of roles vary over time and from region to region, the situation in which the language is used impacts the meaning of the words used, making language an integral part of the learning process (Vrasidas, 2000). By sharing narratives, each student’s level of understanding grows and creates a stronger narrative within each student.

**Critical Pedagogy in the Mathematics Classroom**

Critical pedagogies form a link between classroom experience and the surrounding sociopolitical community. Constructivism acknowledges that all learning is re-learning and that meaning making is an ongoing, socially meditated process (Brown, 2009). According to Dewey (1938)

> Every experience both takes up something from those which have gone before and modifies in some way the quality of those which come after…What he has learned in the way of knowledge and skill in one situation becomes an instrument of understanding and dealing effectively with the situations which follow. (p. 35)

Pairing social constructivism with critical pedagogy leads to significant implications for learning when teaching for social justice by placing reflection and action in the hands of the students.

Often considered a gateway, mathematics is normally thought of as a means of empowerment. However, it can also be a means to oppression, acting as a gatekeeper by selecting who and who cannot participate. Recent findings from the Organization for Economic Cooperation and Development (OECD) support this belief. Every three years OECD administers the *Program for International Student Assessment* (PISA) which measures the literacy of 15-year-old students across the world in math, science, and reading. While all three are indicators of participation in higher education, the OECD has found that, above all other measurements, numerical literacy has the most significant impact on future employment, being directly proportional to employment rates (Andrade-Molina, 2017). Higher educational attainment and the ability to maintain long-term employment increases wealth and is correlated with increased overall well-being, resulting in healthier, happier individuals who live longer (Cutler & Lleras-Muney, 2008). Furthermore, highly skilled people are more likely to volunteer, have higher levels of civic engagement, and are more of trusting others suggesting that "fairness, integrity and inclusiveness in public policy thus all hinge on the skills of citizens" (OECD, 2014). A population lacking numerical literacy not only ensures the existence of a lower economic class, it maintains "the social order in such a 'smart' form that 'rational' citizens, by using their own free will, accept an imposed order" (Lesser & Blake, 2007; Skovmose, 2004, p. 3).

A critical mathematics education is necessary because math education is critical (Skovmose, 2004). A critical mathematics pedagogy is a critical pedagogy that explicitly uses mathematics as a tool to uncover social and political injustices through carefully constructed collaborative learning experiences that allow students to examine, apply, discuss, and reflect upon content (Frankenstein, 1990; Nagada et al., 2003; Stinson, Bidwell, & Powell, 2012). This process helps to foster a critical consciousness, helping students to see how their experiences are situated in historical, cultural, and social contexts, highlighting unfair practices, and possibly leading to policy change (Leonard, Brooks, Barnes-Johnson, & Berry, 2010; Nagada et al., 2003). Because of this, this type of critical mathematics pedagogy is often referred to as teaching mathematics for social justice (TMfSJ). Skovsmose, Frankenstein, and Gutstein are the three key scholars of TMfSJ. While each offers their own unique perspective, they share one common theme: literacy.

As Freire claimed that literacy is more than the ability to read and write, Skovsmose believed that mathematics encompassed much more than the ability to calculate. It is from this notion he developed his idea of *mathemacy* (Skovsmose, 1994). In *Towards a Critical Mathematics Education*, Skovsmose (1994) identifies the three types of knowing on which mathemacy is created from: mathematical knowing, technological knowing, and reflective knowing. Ultimately, the ways of knowing represent the 'how, what, and why' of mathematics. Mathematical knowing, or 'how do I do this?', refers to understanding basic math skills. Its primary concern is if students would be able to recreate theorems or apply algorithms. Technological knowing, or 'what do I use?', measures a student's ability to use technologically based tools when solving problems. While each generation is presented with new technologies and tools, recent advancements in computers and handheld devices, such as graphing calculators, have become an integral part of the mathematics classroom. Often students are required to use these devices to complete assignments, making technological knowledge an integral part of a student's overall success in the mathematics classroom. Reflective knowing, or 'why is this important?', examines a student's competence in reflecting on and evaluating the mathematics used (Skovsmose, 1994).

While all three ways of knowing forms one's mathemacy, it was reflective knowing that was of particular importance to Skovsmose (1994). It is reflective knowing that makes mathemacy critical. Reflective knowing offers a mathematical "checks and balances" on the two previous ways of knowing. One must be able to assess if the correct method was used, the accuracy of the results, and what can be inferred from the findings. Mathemacy, an integrated competency implementing intention, reflection and critique, creates a way to interpret, evaluate, and understand the "formatting power of mathematics", or the way the social structures are created and supported by mathematics (Skovsmose, 1998, p. 45; Skovsmose, 2004). One with a well-developed mathemacy has the means to evaluate, or re-evaluate, interpretations of social institutions, traditions, and proposals for political reforms (Bartell, 2013; Skovsmose, 1994).

Frankenstein's critical mathematical literacy (CML) curriculum focused on using mathematics “to reveal and explode” statistical data for a deeper understanding of issues (Frankenstein, 1990, p. 336). CML is defined as the ability to ask statistical questions in order to develop a deeper understanding or appreciation for socio-political issues, as well as the ability to present data to influence other’s perceptions of the issues (Brown, 2009; Frankenstein, 1990). Like Skovsmose, CML curriculum relies heavily on the work of Freire and his concept of critical knowledge. Frankenstein extends that concept to include not only that statistics is not neutral, but also whose interests are best served by using the statistics (Frankenstein, 1990).

However, CML is also influenced by Giroux and Apple. These influences move CML closer to the TMfSJ framework than mathemacy. Specifically, Frankenstein incorporates Giroux’s formulation of the dialectic and Apple’s analysis of labeling. Formulation of the dialectic has four categories: totality, mediation, appropriation, and transcendence (Frankenstein, 1983).  Totality adds the need to understand facts in a historical, socio-economic, political, and cultural context. This leads to mediation, or the challenging of taken-for-granted assumptions about society and its structures. Challenging these assumptions creates a sense of agency, or appropriation, as we may then begin to transform our worlds. Last, transcendence both completes and restarts the cycle as we now aware we do not have to accept domination and can reconstruct society without unjust structures (Frankenstein, 1983).

Apple’s analysis of labeling adds to Giroux’s theory by exploring the language used by and of the oppressed. Labels sort people into broad categories, simplifying situations and ignoring (or hiding) more complex social, economic, or cultural factors. The label then turns into a brand, with solutions being focused on the individual or group, not the contributing factors. In education, labels relating to tracking, such as ‘non-college bound’ or ‘non-traditional,’ or those related to subjects, like ‘math anxiety’, have shown to result in self-depreciation; once labels are accepted, people often participate and further their own disempowerment (Frankenstein & Powell, 1989; Lesser & Blake, 2007). The labels can be exploited as a means of social control, under the disguise of “natural talent”; people are more easily oppressed when they cannot decode numerical lies and misrepresentations (Apple, 1992; Lesser & Blake, 2007). CML challenges students to question these hegemonic ideologies by using statistics to reveal the contradictions (Frankenstein, 1983).

Gutstein (2006) offers the most comprehensive approach for teaching mathematics for social justice (TMfSJ). TMfSJ has two related pedagogical goals with three subcategories. The first two subcategories under the social justice pedagogical goals build upon Freire’s definitions of praxis, later simplified to reading and writing the world. ‘Reading the world’ is understanding the sociopolitical, cultural-historical conditions of one’s life, community, society, and world (Freire, 1970/2005; Gutstein, 2006). To read the world with mathematics means using mathematics, of any branch, to understand relations of power, inequities, and disparate opportunities between different social groups, to understand explicit discrimination, and to evaluate forms of representation of mathematical data and information (Gutstein, 2003).

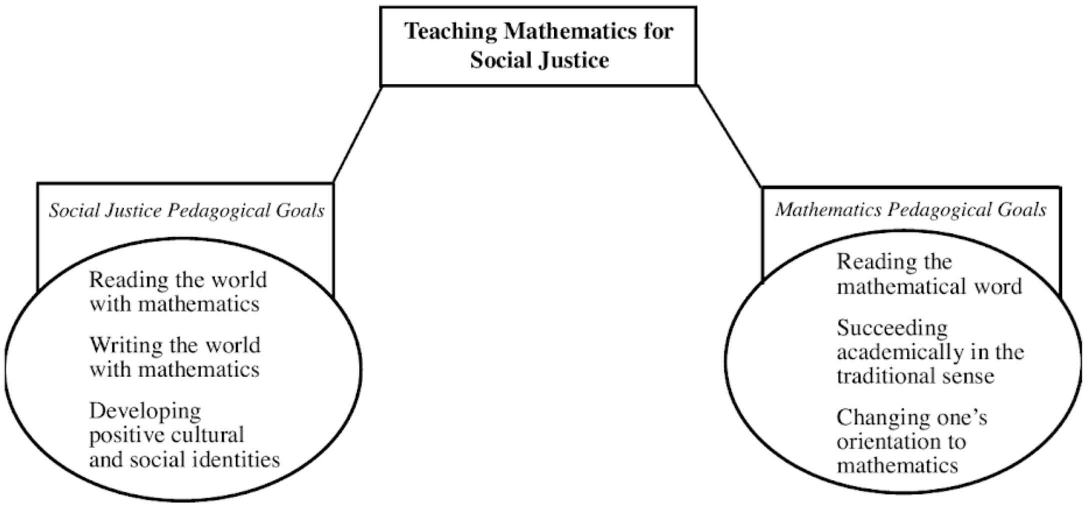



*Figure 1.* Teaching Mathematics for Social Justice (TMfSJ). Reprinted from *Reading and writing the world with mathematics: Toward a pedagogy for social justice* (p. 24), by E. Gutstein, 2006, New York: Taylor & Francis. Copyright 2006 by Taylor & Francis.

'Writing the world' speaks to the action piece of praxis. It is the writing, or rewriting, of what students read in the world due to a developed sense of agency and seeing oneself as being able to make change. Writing the world with mathematics is to create change using mathematics to support, defend, or challenge ways of being or knowing. The third subcategory of developing positive cultural and social identities addresses the development of Ladson-Billing's cultural competence (1995). Cultural competence refers to students being able to maintain cultural integrity while pursuing academic success. Grounding the mathematics in the student's community means they do not need to abandon their cultural identities to successfully gain whatever knowledge is required from the dominant culture of the area (Gutstein, 2006; Ladson-Billings, 1995; Leonard et al., 2010; Stinson, Bidwell, & Powell, 2012).

We see the influence of Ladson-Billing, Skovsmose, and Frankenstein through the mathematical pedagogical goals of reading the mathematical word, succeeding academically, and changing one's view of mathematics. While these subcategories directly relate to the learning of mathematics, they are framed by and with social justice issues and all, in some way, help develop ‘mathematical power’. According to Gutstein (2006), it is the first subcategory of reading the mathematical word that is solely responsible for the creation of mathematical power. Mathematical power is the ability to identify the need for and the ability to apply mathematics to construct ways of solving nonroutine, complex problems and the awareness that mathematics is a tool for addressing sociopolitical issues (Gutstein, 2006; Romberg, 1992; Stinson et al., 2012). The development of mathematical power is critical in TMfSJ. By increasing one’s mathematical comprehension, i.e. – mathemacy, you ‘open the gate’ to advanced opportunities, both in and out of educational settings. It also allows one to make informed decisions by analyzing situations modeled with mathematics, instead of being coerced into having a ‘blind trust’ in numbers (Gutstein, 2006; Skosvmose, 2004).

Often increased mathematical power (MP) leads to the second subcategory of traditional, academic success, but MP is not the only deciding factor in that success. Classroom climate and diversity of the student body is also significant as “classrooms that do not feel inclusive to all students is [a result of ] the lack of retention of students from a diverse group into a particular field” (Ruggs & Hebl, 2012, p. 4). Helping students from diverse backgrounds see a place for themselves in the mathematical community is the first step to increasing student involvement in mathematics. Schools that implement more equitable curriculums, such as TMfSJ, often increase a student’s ability to achieve academic success and combat marginalization and exclusion (Gutstein, 2006; Romberg, 1992; Schoenfeld, 2002).

Curriculum reform leads to the last subcategory under mathematical pedagogical goals. Changing one’s orientation to mathematics begins with the elimination of memorizing rules and worksheets of practice problems. Instruction that emphasizes the interrelatedness of mathematical ideas, students learn not only mathematics but also about the utility of mathematics for understanding the real-world (NCTM, 2000; Stinson et al., 2012). When students are analyzing issues that impact their lives, the information becomes more relevant and they are then less likely to view the material as "a disconnected collection of theorems and plug-and-chug recipes" (Lesser, 2007, p. 7). TMfSJ helps students to create connections between mathematical topics, to other subjects, and in their own lives, adding depth and power to the curriculum. Additionally, use of mathematics' analytical reasoning and tools to explore specific, concrete real-life issues of social justice, their mathematical power is once again increased as they will learn to "…investigate and critique injustice, and to challenge, in words and actions, oppressive structures and acts…" (Gutstein, 2006, p. 4).

**Impact of the Use of Critical Pedagogies**

Use of a critical pedagogy with an emphasis on social justice issues has shown to raise student engagement in both statistics and mathematics courses through reflective inquiry (Cheng, Ferris, & Perolio, 2018; Leonard et al., 2015; Lesser, 2007; Voss & Rickards, 2016; Wright, 2016). As "all reflective inquiry starts from a problematic situation" (Dewey, 1929/2008, p. 181) the problematizing of the curriculum with social justice issues from their daily lives requires students to examine both the context and the content being presented. Through reflective inquiry students discover the importance and usefulness of the mathematics and statistics, developing a more positive impression of the subject (Hiebert et al., 1996; Voss & Rickards, 2016; Wright, 2016).

In the mathematics classroom, critical pedagogy is shaped by the PSSM, even at post-secondary levels. A curriculum that values communication, collaboration, and reflection, the PSSM aims to provide all students the chance to gain mathematical competence. It is PSSM's five process standards of problem solving, reasoning and proof, communication, connections, and representations that support the use of critical pedagogy in a mathematics classroom and exemplify a commitment to mathematical literacy. Schoenfeld (2002) reported findings of a large-scale, district-wide initiative to implement the standards proposed in the PSSM to over 40,000 students in 97 public schools in Pittsburgh. Schools that reformed their curriculums using the PSSM guidelines significantly increased the number of students meeting or exceeding standards and racial differences in performance decreased.

In 1997, 10% of students in Pittsburgh schools met or exceeded standards for concepts or problem solving, but after the implementation of the PSSM that rose to 25%. On the skill standard, prior to the PSSM, less than a third met or exceeded standards. After the reform, nearly 60% did. To examine the impact of equity, a matched-pairs sample of individual schools based on socioeconomic status (SES) was formed. Schools were classified as either strong implementation or weak implementation of the reform. Students at the strong implementation schools performed at significantly higher rates than students at the weak implementation schools, the percentage of African-American students meeting each of the concepts and problem solving standards was 30% or more, and the ratio of White students to African-American students who met standards decreased from four to one to three to two (Schoenfeld, 2002). Michigan and Massachusetts showed similar results. After Michigan realigned with the PSSM, eighth graders in a sample group of low-SES districts scored higher than other states that participated in the Third International Mathematics and Science Study–Repeat. In Massachusetts, fourth grade and eighth grade students using PSSM materials outperformed matched comparison groups who continued with traditional textbooks commonly used in the state (Schoenfeld, 2002).

Despite the guidance and effectiveness of the PSSM, research suggests that the use of critical mathematics pedagogies, such as Teaching Math for Social Justice (TMfSJ), is uncommon due to lack of training and resources (Leonard & Moore, 2014; Ukpokodu, 2007). When asked about TMfSJ, participants in Ukpokodu's (2011) study expressed that these were new concepts to them and that they had never seen or heard of them before. The college courses they had taken did not discuss the use of critical pedagogies or model them, so they did not know what such instruction looked like. Harrison (2015) in her self-study acknowledged that, "My lack of preparation in conceptualizing social justice came out as a weakness in my ability to successfully teach mathematics for social justice" (p. 6). Gonzalez (2009) found that the participants in her study met the idea of TMfSJ with both interest and caution as "the teachers worried if raising awareness about social issues would serve to paralyze rather than motivate students…[and] that teaching mathematics for social justice might not be supported by the school's administration or by parents" (p. 47).

When modeled and provided with resources, teachers appear to be receptive to teaching for social justice. A study by Leonard and Moore (2014) with 23 teacher candidates (TC) revealed promising, though not surprising, data. The participants were in enrolled in an eight-week mathematics education course as part of an initial licensure program. The results of this study revealed two important findings. First, TCs in the study were successful in implementing their own social justice-oriented mathematics lessons after seeing teaching examples. Second, understanding the importance of student outcomes helped to influence TCs' beliefs about how TMfSJ increased students' empowerment and agency (Leonard & Moore, 2014). These are important findings as teaching for social justice has shown to better meet the needs of a diverse student body and close the achievement gap (Brown, 2009; Frankenstein, 1983; Gutstein, 2003; Lesser, 2007; Ruggs & Hebl; 2012; Ukpokodu, 2011; Winter, 2007).

Increasing a student's success in the classroom increases their confidence in their ability to complete mathematically based tasks both in and out of the classroom (Ernest, 2002). Examples of this have been found in a variety of educational levels and topic areas. Gutstein (2003) completed a two-year qualitative study about teaching and learning mathematics for social justice in an urban, Latino classroom. The study revealed students connected mathematical ideas to their growing understanding of the socio-political context of society after the use of TMfSJ. All passed their eighth-grade standardized tests and students gained one month on their standardized test scores for every month they were in the course. Additionally, 15 of the 18 students who took entry exams in mathematics and language arts for magnet high schools were accepted.

Winter (2007) created a project for an undergraduate Pre-Calculus class that used the context of water rights in Botswana to teach the concepts of piecewise defined functions, domain, and range instead of using worksheets and textbook examples. Winter's (2007) study found "The percentage of unsuccessful students in the experimental group was 15.4 percent, which was significantly…lower than the 22.9 percent of unsuccessful students in the control group" (p. 104). The students in one class were so moved by their findings that they began a fundraiser to help raise money to donate to a charity that aids in the water rights movement.

Frankenstein (1990) created several active learning activities that implemented TMfSJ by selecting examples that used real-world data and involved open-ended questions, such as comparing the median incomes of men and women and examining the possible impact of incorporation of a city in Massachusetts. This was necessary as, "real life is messy, with many problems intersecting and interacting. Real life poses problems with solutions that require dialogue and collective action" (Frankenstein, 2013, p. 32). Grappling with messy data and multilayered problems raises students' mathematical abilities and challenges the one-right-answer narrative that is pervasive in mathematics education. Students must truly learn to "read the world" with mathematics to create solutions and change their orientation with mathematics from being a set of disconnected rules to a powerful tool (Gutstein, 2006). By challenging taken-for-granted assumptions and making connections, students increase their ability to read the mathematical world.

Once a critical consciousness is developed and one learns to 'read and write the world with mathematics', the belief that mathematics is value-free or neutral quickly fades (Gutstein, 2006). There is no such thing as a neutral educational process; no knowledge or teaching can be neutral or apolitical (Freire, 1970/2005; Leonard & Moore, 2014; Leonard et. al, 2010; Stinson, 2004; Tutak et al., 2011). Mathematics is used to fly drones during military missions, devise financial derivatives, develop tools to help the visually impaired, advance HIV/AIDS research, and express the dangers of fossil fuels (Gutstein & Peterson, 2013). Once students learn that politically-free or "true" mathematics is false, mathematics becomes a discipline of empowerment and inclusion instead of a discipline of oppressive exclusion (Stinson, 2004).

The non-neutrality of mathematics can easily be seen in the applied mathematical field of statistics. Statistics and other mathematical representations reflect choices and are not neutral, value-free records of the world; data cannot exist independent from how and why it is used (Frankenstein, 1983; Weiland, 2017). The sheer act of counting, or collecting data, is not neutral as "it serves as a way to quantify objects, in some cases to determine value…" (Leonard et al., 2010, p. 262). What is valuable to one group may not be to another, impacting what does and does not get counted. Using statistics to identify group differences or patterns in data related to socioeconomic scenarios can help to expose social injustices and spur change. By analyzing data that impacts a student's personal life and community, students become aware that mathematics is a powerful tool, analytically and politically. As assertions are generally deemed more credible when supported by quantitative data, statistics has become the language of persuasion (Brelias, 2015).

**Focus on Community Colleges**

While the Obama administration had adopted a goal of 8 million more postsecondary degrees by 2020, they also specifically called for 5 million additional community college graduates, despite the fact in 2009 twice as many baccalaureate degrees were granted than associate level ones (Bailey, 2012). The completion agenda presents two serious, unique issues for community colleges. The more innocuous of the two is the definition of completion. Graduation rates for community colleges are calculated by tracking a cohort of first-time, full-time students for three years, but these rates only track students through the college at which they enrolled first. 'Early transfers', students who did not complete a pathway, but transfer to another college, are counted as "noncompleters," regardless if they complete their degree at the next institution (Bailey, 2012). While stronger articulation agreements were believed to be a solution for discouraging 'early transfer' students, no relationship between statewide articulation policies and transfer rates from community colleges to four-year institutions has been found (Roksa, 2013). Therefore, an 'early transfer' student that completes a pathway at the next institution has a negative impact on the community college's graduation rate. Additionally, the institution that the student transferred to is not able to count the student as a 'completer' since they did not begin their studies at that school.

For those who do complete a pathway, there are discrepancies about which pathways count. The goal set by the Bill and Melinda Gates Foundation included "only degrees with "genuine labor market value" for individuals up to 26 years old; the administration's goals refer to 25- to 34-year-olds, and Lumina's, to 25- to 64-year-olds…complications still arise concerning the definitions of 'additional' and 'degrees.'" (Bailey, 2012, p. 77). There is also a matter of declining enrollment at community colleges due to an improving economy. Assuming that 'degrees' refers to degrees and certificates that take at least one year and 'additional' degrees would be those that surpassed predictions based upon data collected before 2009, community colleges would need to maintain a growth rate in enrollment of approximately 8.5% a year in order to award the number of degrees required to meet the goal (Bailey, 2012).

The more pressing issue of the completion agenda for community colleges being asked to shoulder more of the burden of the 2020 vision relates to the population they serve, not to population size. Community college students are more likely to be underprepared for college level work, be considered non-traditional students, and live in poverty (Bahr, 2010; Brock, 2010; Horn, Nevill, & Griffith, 2006; Kolesniskova, 2009; Quarles & Davis, 2017). Students from the lowest socioeconomic status (SES) not only have lower rates of persistence and degree completion, but they often have lower educational aspirations. Walpole (2003) notes this is a result of variety of influences, such as parental definitions of success, K-12 school experiences, and overall college costs.

Those living in the lowest SES are more likely to be Black or Hispanic (Musu-Gillette et al., 2017; Pew Research Center, 2016). Overall, as of 2006, the population of undergraduate students has become more diverse as the proportion of Hispanic students has tripled, the proportion of Asian students nearly quadrupled, the proportion of black students has increased by 40%, and more than half of all undergraduates are female (Brock, 2010; Goldrick-Rab & Cook, 2011). While access to higher education has clearly improved, Brock (2010) has noted the ‘center of gravity’ in higher education has shifted to community colleges, as “female, Black, and Hispanic students are disproportionately enrolled at community colleges” (pp. 113-4). Graduation rates have been found to be closely related to race and SES as “29.5 percent of white community college entrants complete a bachelor’s or associate degree, only 16.5 percent of black entrants do. The disparity is only slightly less for Hispanic students” (Bailey, 2012, p. 85).

Sixty-three percent of students from the bottom socioeconomic quintile will need remediation (Goldrick-Rab & Cook, 2011). Because of this, community colleges must be more than an “open door”. Community colleges must also provide remedial courses as “a lifeline in the ascent to financial and social-structural stability for individuals who face significant deficiencies in foundational subjects” (Bahr, 2010, p. 209). Most often, the individuals are Black and Hispanic students and the foundational subject is mathematics. While nearly 60% of community college students will require at least one remedial course, Blacks and Hispanics are once again disproportionally represented with 62% of Blacks and 63% of Hispanics requiring remediation compared to 36% of Whites and 38% of Asians (Bahr, 2010; Bailey, 2012).

Racial gaps in educational achievement are seen as early as Kindergarten in mathematics (Bahr, 2010). These gaps widened over time and create racial and socioeconomic stratification. Due to these educational gaps, remedial education often acts as a gatekeeper instead of a bridge to college-level mathematics (Brock, 2010). Placement in either remedial or college-level mathematics is largely determined by a standardized exam and disproportionately impacts minority students (Attewell, Lavin, Domina, & Levey, 2006; Bahr, 2010; Walker & Plata, 2000). Additionally, two-year college students are more likely to place into a remedial course than four-year colleges students are, “even for students with equivalent academic skills and background” (Attewell et al., 2006, p. 902).

There is a silver lining to the seemingly dark cloud of remediation. Bahr (2010) found that students who successfully complete their required remedial mathematics sequence are as likely to transfer to a four-year university as those who did not require remediation. Additionally, “remedial completers” are more likely to transfer with a completed pathway than those who do not require remediation (Roksa, 2013). This is of particular significance as Kopko and Crosta (2016) found that associate degree holders who transferred are 50% more likely to complete a bachelor’s degree in six years. As approximately 17% of all bachelor’s degree holders first identified as first completing an associate degree (Kolesnikova, 2009), improving remedial mathematics success rates can potentially improve retention.

One of the most effective strategies for increasing success in remedial mathematics courses has been course redesigns. Course redesign is the act of realigning entire courses, not just sections or classes, with the goal of improving student success. Currently, remedial mathematics course redesigns in North Carolina have heavily focused on modularization, acceleration, supplementary instruction, and use of technology but in the 2020-2021 academic year the focus will shift to mainstreaming and the use of multiple measures. ‘Multiple Measures’ (MM) allows students who meet designated SAT or ACT math scores or have completed specific high school level mathematics sequences with overall high school GPAs over 2.8 to bypass placement testing and enter curriculum level mathematics courses. MM implements a five-year limit on test scores and high school GPAs as knowledge retention greatly reduces over time (Garner & Garner, 2001; Kwon, Rasmussen, & Allen, 2005).

Mainstreaming offers supplemental instruction to students while they are enrolled in the college-level course instead of requiring a remedial course and has shown to be successful for remedial English and Basic Skills (Brock, 2010). A large study completed at three CUNY community colleges found that students who were placed in a statistics course with supplemental instruction passed with similar rates to those who completed the traditional track. However, while pass rates were slightly lower overall (68% traditional as to 55% mainstreamed), it showed a positive impact on 'academic momentum' (Adelman, 2006).

> To complete statistics within two semesters of entry, such students would have to pass elementary algebra in the fall (the actual pass rate is 37%), return in the spring (the overall retention rate for freshmen from fall to spring is 84%), and then take and pass statistics in the spring (CUNY statistics students who have previously passed elementary algebra have a 68% statistics pass rate). …the probability of completing statistics within two semesters for these students is therefore only .37 times .68, that is, .25. In contrast, 55.69% of the Stat-WS participants passed statistics in their first semester…(Logue, Wanatabe-Rose, & Douglas, 2016, p. 592)

Austin Peay State University in Tennessee found similar results in 2007. Through a grant from the U.S. Department of Education to implement the Developmental Studies Redesign Project, four institutions aspired to improve the effectiveness of their remedial courses. At Austin Peay State, they restructured two core college-level mathematics courses, Fundamentals of Algebra and Elements of Statistics, to include Learning Assistance Workshops that provide tutoring and assistance. Students whose ACT scores placed them in remedial mathematics were instead enrolled in one of the two redesigned courses. Of the four redesigns implemented, Austin Peay State University saw the largest gains on persistence (Bettinger, Boatman, & Long, 2013).

The emphasis on numerical literacy has been growing the last twenty years (Ben-Zvi & Garfield, 2008). The idea of numerical literacy has numerous synonyms, often also being referred to as quantitative literacy (QL), quantitative reasoning, and numeracy. However, regardless of the term, it is consistently listed by educators as one of the most important outcomes of a liberal arts education (Schield, 2005). Algebra and Calculus, however, are never mentioned causing leaders in the QL movement to call for a reform of mathematics education, particularly for non-STEM majors. Traditional mathematics courses emphasize the procedure for calculating solutions and does little to prepare students for a “data-drenched,” computer-based world (Erickson, 2016). Steen (2001), a leader in the QL movement, states “virtually every major public issue --from health care to social security, from international economics to welfare reform-- depends on data, projections, inferences, and the kind of systematic thinking that is at the heart of quantitative literacy” (p. 10). As QL is “anchored in context” and its objects are data, not ideas (Steen, 2004, p. 5), it finds itself at “the intersection of statistics, mathematics, and democracy” (Steen, 2004, p. 62). With a focus on the ability to evaluate and synthesize numerical information, statistics classes continue to gain more students each year, particularly in community colleges (Blair et al., 2015).

Due to the QL movement, and an estimated 50% of college graduates taking a statistics course, the most recent North Carolina Pathway redesign emphasizes the study of statistics in the remedial mathematics redesign as it can be used to teach the critical skills needed in the workplace and to create informed citizens (Lesser, 2007; Schield, 2005; Tishkovskaya & Lancaster, 2012). About 42% of first-year undergraduates at two-year public institutions enroll in at least one remedial course (Clotfelter, Ladd, Muschkin, & Vidgor, 2015). As the number of required prerequisites increase, the road to completion gets longer, lowering the probability of degree attainment (Clotfelter et al., 2015). To reduce the pathway to graduation, there is a new reform effort, Reinforced Instruction for Student Excellence (RISE), that focuses on the mainstreaming of remedial courses (North Carolina Association for Developmental Education, 2017). Instead of taking remedial mathematics courses as prerequisites, students will be required to take a one credit co-requisite course in addition to the curriculum mathematics course. There are three mathematics transfer courses impacted by RISE, MAT 143, MAT 152, and MAT 171. MAT 143 and MAT 152 heavily emphasize the study of statistics. As critical pedagogies offer a multi-tiered approach to helping students acquire the math credits required for graduation, this study was designed to specifically explore the benefits of implementing a critical pedagogy in a community college statistics classroom.

**Teaching Statistics for Social Justice**

There are many different fields of study in mathematics, some are referred to as pure and others as applied. Pure mathematical thinking focuses on abstract patterns and "the context is part of the irrelevant detail that must be boiled off over the flame of abstraction in order to reveal the previously hidden crystal of pure structure…context obscures structure" (Cobb & Moore, 1997, p. 803). However, it is the opposite with applied mathematics, like the field of statistics, where the context provides meaning. Applied mathematics is roughly defined as the application of mathematics where the focus is on the use of the mathematics, not on the mathematical theories being applied (Dörfler & Mclone, 1986). As a statistic is a number *with* context, the field of statistics is an applied mathematics, causing statistical thinking and mathematical thinking to differ (Cobb & Moore, 1997; Lesser, 2007).

While the two inherently overlap, the curriculum goals and pedagogical approaches between teaching pure and applied fields do vary. In the *Guidelines for Assessment and Instruction in Statistics Education College Report* (GAISE), the American Statistical Association (ASA) offer six recommendations and nine learning goals for creating statistically literate students (Carver et al., 2016). Statistical literacy is the ability to discuss one's understandings of and reactions to the data and to communicate both the conclusions and the concerns they may raise (Tishkovskaya & Lancaster, 2012). Like Skovsmose's *mathemacy*, ASA's recommendations and learning goals call for statistical, technological, and reflective learning. The recommendations encourage educators to teach statistics as an investigative process of problem-solving and decision making, to integrate real data with context and purpose, focus on conceptual understanding, foster active learning, and use technology to explore concepts and analyze data.

The learning goals support and expand on those recommendations. In addition to suggesting instruction of particular statistical concepts, like understanding randomness and calculating confidence intervals, and use of statistical software, five of the goals specifically focus on reflective knowing by requiring students to 1) be critical consumers of statistically based reported results, 2) question the usefulness of investigative processes in statistics, 3) produce and interpret graphical displays and numerical summaries, 4) understand and apply statistical inference, and 5) demonstrate an awareness of ethical issues associated with sound statistical practice (Carver et al., 2016). To achieve this level of statistical literacy, in addition to becoming statistically competent with the concepts and tools of statistical reasoning and thinking, students will need to become 'statistical citizens' and develop a critical understanding of the data in order to act from a more informed position when analyzing ethical issues (Frankenstein & Powell, 1989; Lesser & Blake, 2007; Rumsey, 2002).

The GAISE recommendations and learning goals easily lend to the use of a critical pedagogy as they facilitate critical thinking, require active learning, and aim to challenge students' biases and assumptions (Nagada et al., 2003). Lesser's (2007) Teaching Statistics using Social Justice (TSSJ) is a critical statistical pedagogy that is defined as the teaching of statistics using real world examples related to social justice to enable and empower students to use statistics to ‘talk back’ to the world. Largely founded on Gutstein’s (2006) TMfSJ and GAISE recommendations and learning goals,

> TSSJ can be viewed as a way of teaching statistics that includes the conceptual and computational proficiency goals recommended by most leading statistics educators, but also has a critical perspective that incorporates and facilitates awareness of issues of social justice and prepares students not only to be competitive workers in the economy but also engaged participants in a democracy, able to be critically reflective about the role statistics has played and can play in our society. In other words, statistics must be seen not merely as useful (for working, shopping, etc.) but also as a tool to help effect social change in the world. (Lesser, 2007, p. 3)

Weiland’s (2017) critical statistical literacy complements Lesser’s (2007) work by specifically defining what it means to be critically statistically literate. Framed by the pedagogies previously explored, the critical statistical literacy (CSL) perspective emphasizes the importance of critically ‘reading and writing the world’ using statistics to explore sociopolitical contexts (Weiland, 2017). “Reading the world” in CSL includes understanding statistical language and symbols, establishing a statistical way of knowing. Including the need to identify and question data-based arguments of social structures and the evaluation of statistical sources through a social, historical, and political lens adds reflective ways of knowing, for a more critical reading of the world. The same is true for “writing the world” in CSL where one not only communicates the statistical data, but also how one’s position impacts their “overall meaning making of the world” and use of statistics to investigate and response to unjust structures (Weiland, 2017, p. 42).

As this study was completed in an introductory statistics classroom, the conceptual framework of Teaching Statistics for Social Justice (TSfSJ) that guided the study was founded upon Gutstein’s (2006) TMfSJ, Lesser’s (2007) TSSJ, and Weiland’s (2017) CSL. Teaching Statistics for Social Justice (TSfSJ) is a critical statistical pedagogy designed for introductory statistics classrooms as it assumes students do not have prior knowledge or experience using statistics, examining and discussing social justice issues, or implementing a critical lens. Class discussion and data sets consisted of real-world, local, nontrivial examples for students to both learn and reflect upon the context of these examples and learn how to use and apply course content (Lesser, 2007). TSfSJ was designed with the following outcomes in mind:

1. increase mathematical empowerment by increasing confidence in their ability to complete statistical procedures that produce results relevant to the surrounding community;
2. instill a sense of social justice to encourage civic engagement, activate critical voice, and increase personal relevance, and;
3. increase student engagement through data sets that examine important social issues that are relevant to their lives to increase student success.

Students were expected to use statistical methods to read and write their world using predetermined data sets examining social justice issues surrounding race, class, gender, and sexual orientation. Through reflective inquiry students discovered the importance and usefulness of statistics, developing a more positive impression of the subject (Hiebert et al., 1996; Voss & Rickards, 2016; Wright, 2016).

Reading the world with statistics included "identifying and interrogating social structures and discourses that shape and are reinforced by the data-based arguments" (Weiland, 2017, p. 41) with a particular focus on power, inequities, and disparities between different social groups (Gutstein, 2003). Students then examined these arguments, via statistical analysis and class discussion, to discover what, if any, injustices are being perpetrated, uncover possible inconsistencies, determine whether to accept or reject the arguments, and provide support using statistical investigations to communicate statistical information and arguments (Carver et al., 2016; Gutstein, 2003;  Lesser, 2007; Weiland, 2017).

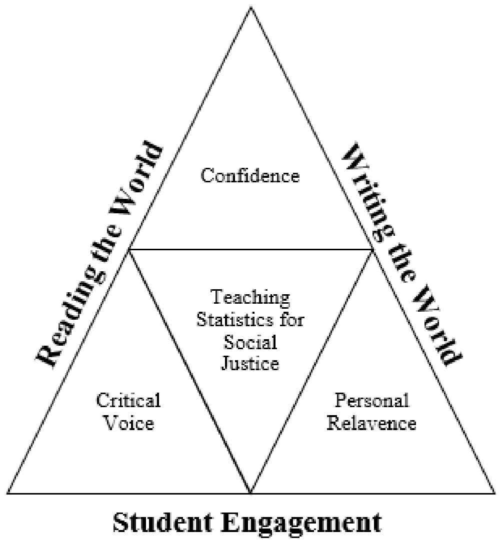



*Figure 2.* Teaching Statistics for Social Justice (TSfSJ).

Writing the world for statistics was restricted to making sense of and critiquing statistical and quantitative data-based arguments and evaluating the source, collection, and reporting of statistical information. Students were encouraged to offer counternarratives, expand upon the data sets provided with their research when drawing conclusions, and examine sources for both legitimacy and for missing data. Ultimately, TSfSJ is a critical statistical pedagogy that aims to ensure students learn to critically evaluate data and comprise evidence-based arguments to both better retain content, improve course success, and become more informed, empowered citizens.

# Chapter Three: Methodology

This chapter focuses on discussing the research design and describing the participants. Then the role of the researcher and ethical issues are examined. Next, data sources and means of collection are identified. Last, how the data was utilized to measure the use of the critical pedagogy TSfSJ in the undergraduate statistics classroom in a community college will be shared. The following research questions guided the study:

1. Does implementing a critical pedagogy increase student success? Student success is defined as completing the course with grade of a C or higher.
2. Does use of a critical pedagogy increase the student's sense of mathematical empowerment? MP is measured using a subset of questions from the Constructivist Learning Environment Survey.

## Research Design

Quasi-experimental designs meet the following three requirements: there must be a treated and untreated group, a pre-treatment and/or post-treatment measure must be administered, and an explicit model must exist that projects the difference between the treated and untreated groups (Kenny, 1975). Nonequivalent control group design (NCGD) studies are one of the most common, and recommended, designs in educational research as they allow researchers to use preformed classrooms for the treatment and control groups (Campbell & Stanley,1963; Dimitrov & Rumill, Jr., 2003; Steiner, Wroblewski, & Cook, 2009). The use of preformed groups, formed either by self-selection or nonrandom assignment by administrators, prohibits a NCGD from being considered a true experiment. As the participants in a NCGD are not assigned to groups at random, the participants could differ systematically even before the treatment is applied (Reichardt, 2009). Use of a pretest allows the researchers to 'control by design’ through testing and examination of biases that often threaten NCGDs (Christensen, Johnson, & Turner, 2014). Figure 2 represents a NCGD with X representing the treatment.

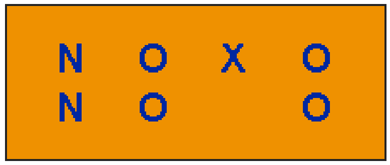



*Figure 3*. NCGD. Adapted from The Nonequivalent Groups Design, by W.K Trochim (2006). Retrieved February 2, 2019 from http://www.socialresearchmethods.net/kb/quasnegd.php. Copyright 2006 by Trochim.

The purpose of this study was to measure the effectiveness of implementing a critical statistics pedagogy in an undergraduate introductory statistics classroom, and its impact on course success and mathematical empowerment. Intact classroom groups were used as the control and treatment groups, as it was not possible to implement random selection of group members. Groups were randomly assigned as treatment or control. As all groups must meet the same prerequisite skills to enroll in the course, it was reasonable to assume that the groups would be similar. However, as suggested by the research, a pretest/posttest design was used in measuring both research questions to measure preexisting differences, if they existed, before the treatment was administered, adding credibility to the results.

**Participants**

This quasi-experimental study consisted of four MAT 152 Statistical Methods I courses at Mitchell Community College (MCC). During the Fall 2017 semester, MCC had 1,091 full-time students, 2,093 part-time students, and 20 of unknown status. Of the 3,204 students, 35% of students were matriculated in a transfer degree program and 34% were dual-enrolled students either from one of three Early Colleges affiliated with MCC or from one of three local high schools attending college classes through the state’s Career and College Promise program.

Excluding missing data from the 20 students with unknown status, students are primarily white (70%), female (61%), and young with 77% of the students being under 25 years of age. They primarily reside in the cities of Statesville and Mooresville, at 41% and 36% respectively. To best serve their students, MCC has two campuses: Statesville and Mooresville.

Both campuses are "full service" consisting of tutorial services, library, computer labs, financial aid resources and advisors, and administrative offices. Program pathways for the Associate of Arts, Associate of Science, and Applied Associate of Science degrees for Computer Information Technology and Business can be completed on the Mooresville Campus without traveling to the Statesville Campus. Other programs require travel between the two campuses, which are roughly 25 miles apart. Transportation via iCats is provided at pre-posted times each day for a $1 fee each trip (for a cost of $2 for a round trip). Parking is free at both campuses but requires a permit sticker.

| Subject | ZCTA5 28115 | | | | ZCTA5 28117 | | | | ZCTA5 28677 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Total | | Median income (dollars) | | Total | | Median income (dollars) | | Total | | Median income (dollars) | |
| | Estimate | Margin of Error | Estimate | Margin of Error | Estimate | Margin of Error | Estimate | Margin of Error | Estimate | Margin of Error | Estimate | Margin of Error |
| Households | 12,672 | +/-362 | 57,417 | +/-3,406 | 14,226 | +/-351 | 80,126 | +/-3,088 | 12,697 | +/-404 | 40,877 | +/-2,324 |
| One race-- | | | | | | | | | | | | |
| White | 89.50% | +/-1.5 | 59,911 | +/-4,041 | 90.40% | +/-1.0 | 78,983 | +/-3,794 | 72.10% | +/-1.9 | 47,729 | +/-2,599 |
| Black or African American | 7.70% | +/-1.2 | 41,029 | +/-4,367 | 3.10% | +/-0.8 | 63,200 | +/-29,082 | 24.90% | +/-1.8 | 24,949 | +/-3,962 |
| American Indian and Alaska Native | 0.10% | +/-0.1 | - | ** | 0.30% | +/-0.2 | 64,205 | +/-10,516 | 0.10% | +/-0.1 | 61,591 | +/-18,855 |
| Asian | 1.40% | +/-0.5 | 86,510 | +/-29,912 | 5.20% | +/-0.9 | 92,400 | +/-17,402 | 0.30% | +/-0.3 | - | ** |
| Native Hawaiian and Other Pacific Islander | 0.00% | +/-0.3 | - | ** | 0.10% | +/-0.1 | - | ** | 0.00% | +/-0.3 | - | ** |
| Some other race | 0.60% | +/-0.5 | - | ** | 0.90% | +/-0.6 | - | ** | 1.70% | +/-0.7 | 37,273 | +/-27,628 |
| Two or more races | 0.60% | +/-0.5 | 41,136 | +/-24,773 | 0.10% | +/-0.1 | 99,500 | +/-97,267 | 0.90% | +/-0.5 | 31,985 | +/-18,205 |
| | | | | | | | | | | | | |
| Hispanic or Latino origin (of any race) | 4.10% | +/-1.2 | 39,097 | +/-15,129 | 3.80% | +/-1.0 | 38,304 | +/-10,393 | 6.10% | +/-1.3 | 31,219 | +/-8,202 |
| White alone, not Hispanic or Latino | 86.10% | +/-1.7 | 60,970 | +/-3,311 | 87.40% | +/-1.4 | 80,302 | +/-3,145 | 68.10% | +/-2.0 | 47,906 | +/-2,579 |

*Figure 4*. Mooresville and Statesville demographics. Retrieved from American Fact Finder on October 15, 2018. Copyright 2018 by U.S. Census Bureau

The Statesville Campus is in a rural area of Iredell County while the Mooresville Campus is more suburban. While Statesville's 2017 population of 34,333 residents is noticeably smaller than Mooresville's total population of 74,036 (36,679 in zip code 28115 and 37,347 in zip code 28115), the demographics differ greatly among the three areas. The population of Mooresville is roughly 87% white, while Statesville is 69% white. The median incomes between the two cities vary more drastically. Statesville, zip code 28677, has a median income of approximately $41,000. Mooresville, zip codes 28115 and 28117, has median incomes of $57,000 and $80,000 respectively (U.S. Census Bureau, 2018). Other household demographics and differences are noted in Figure 3.

The difference in household income is evident in the high schools. Statesville is served by one public high school, Statesville High. In 2017, Statesville High failed to meet academic growth, incoming student readiness (percent of students considered proficient) was measured at 12.7%, and served a population that was 60% economically disadvantaged ("School Report Card for Statesville High", 2019). Mooresville is served by two public high schools, Lake Norman High and Mooresville High, both of which exceeded academic growth projections in 2017. Lake Norman High had an incoming student readiness of 59.7% and served a population that was 8.9% economically disadvantaged ("School Report Card for Lake Norman High", 2019). Mooresville High's incoming student readiness was 65.9% and its economically disadvantaged population was 30% ("School Report Card for Mooresville High School", 2019). The county's three Early Colleges serve as an additional option for a secondary education. While they do provide bussing and food services, they only accept between 50 and 60 students per year. Additionally, they adhere to a strict behavior policy and academic standard. Students with continuing behavioral issues or failing grades are often reassigned to their zoned schools.

## Role of the Researcher

Ideally in quantitative studies the role of the researcher is nonexistent; it is detached and impartial. By simply adopting to implement a quantitative study, the researcher hints at their own ontological and epistemological assumptions that reality is singular, objective, and measurable. Quantitative studies are conducted using a framework that is “value-free, logical, reductionistic, and deterministic…[and] endorses the view that…the knower or the researcher and the known or subjects are viewed as relatively separate and independent” (Yilmaz, 2013, p. 312). This positivist framework is normally well-developed with identified variables and relationships measured using standardized instruments and/or predetermined response categories with the expectation that all participants’ perspectives and experiences will fit (Punch, 2013; Yilmaz, 2013).

While this study was quantitative, it was also quasi-experimental and primarily did not implement pre-developed materials or a prescribed framework. Materials used consisted of those created solely by the researcher and pre-developed materials that were redesigned for the study. The framework was one that was a collection of overlapping conceptual and theoretical frameworks as described in the previous chapters. Additionally, as this study was completed in an educational setting, the instructor was the researcher for all the courses included in the study. The use of a researcher designed framework, researcher created materials, and the daily interaction of the researcher with the participants, expanded the normally narrow assumptions of quantitative studies to one that is more similar to a qualitative study, where the researcher spends time with participants ‘in the field’ and allows them to openly discuss narratives and interpretations both in the words of the participants and their own (Yilmaz, 2013). As this study used statistics to explore issues of social justice relating to race, class, gender, and sexual orientation, the role of the researcher raised the possibility of ethical concerns**.**

**Ethical Issues**

In quantitative studies, ethical issues are often limited to concerns about withholding resources during experiments and what one can, and cannot, ethically manipulate. However, "the claim that traditional approaches to truth forcibly submerge political and ethical issues is simply wrong" (Cook, Campbell, & Shadish, 2002, p. 483). Maxwell (1992) argues that a singular reality does not imply a singular truth; scientific realism allows for the researcher to assume the world "is as it is" while still acknowledging there are multiple ways to understand reality. People hold multiple, subjective theories of the world and some of these theories better approximate objective reality than others (Smith, 2006). Simply put, according to scientific realism, the role of science is to improve and expand methods of measurement to better separate fact from fiction and generate the most accurate possible description and understanding of the world (Hunt, 1990).

Respect for multiple perspectives and experiences is critical while exploring issues of social justice. Just as research methods have embedded values regarding one's worldview and ways of knowing,

> users of these methods also infuse their investigations with values that render those investigations non-neutral. That is, scientific exploration is never value-free, regardless of methodological approach, but is culturally and temporally situated, enacted by human beings who bring their own unique lenses to each research endeavor. (Fassinger & Morrow, 2013, p. 70).

Social justice issues, by default, are context-driven and are orientated towards creating change. Large gaps often occur between the researcher and the participants of social justice research, with the researcher being deemed as an 'insider' to the dominant culture. In an educational setting, it is common for the researcher to be a white, heterosexual female, possessing privilege through race, class, education, and status (National Center for Educational Statistics, 2018; Taie & Goldring, 2017). Cultural competence, awareness of biases, and an understanding of power and position are critical for researchers that are ‘insiders’ in order to successfully disrupt the status quo instead of perpetrating it (Fassinger & Morrow, 2013).

Quantitative data can provide large, representative samples of a community, reliably assert cause-and-effect relationships, and, most importantly, summarize data in ways that are clear and persuasive to the community and to its leaders and policymakers (Fassinger & Morrow, 2013). By focusing on data that is relevant to the community, quantitative research can expose and help to correct unjust practices (Cokley & Awad, 2013). For this study, all data presented in the researcher created materials used the most recent available data for Iredell County, the county MCC serves, from government databases and surveys and other reputable, independent sources.

The use of local (town, county, district, or state) data provided participants with meaningful data that was a fair, representative sample of their community so that they could explore social justice issues without numerical bias, begin to identify any existing inequities, and confront their own biases. Additionally, after examining all datasets, participants were challenged to answer reflection questions to encourage a deeper dive into the data. These questions included, but were not limited to, “What does the data NOT show?”, “Are there any flaws in the dataset?”, and “What assumptions were made?”. These questions served to further expose any attitudes, biases, and worldviews of both the researcher and the participants.

### Data Sources and Collection

During the Fall 2017 and Spring 2018 semesters, one MAT 152 course on each campus was a part of the NCGD pretest/posttest study. The Institutional Review Board (IRB) at Appalachian State University deemed that the study (17-0357) met the exemption category (1) - *normal educational practices and settings*. The study was determined to involve minimal risk and in accordance to 45 CRF 46.101 (b), the research activities were exempt from further IRB review. However, all participants were made aware of the study and provided with a form detailing the nature of the research and contact information for the dissertation chair and IRB at Appalachian State University. Pretest/Posttest data was collected from the Fall 2017 Statesville section (10 students) and the Mooresville section (19 students). In Spring 2018, pretest/posttest data was collected from 19 students in Statesville and 23 students in Mooresville. Final exam scores and overall course grades were collected from all students who completed the courses. Additional course demographics are listed in Table 1. 'Completer-repeaters' are defined as students who were repeating the course during the study due to either prior withdrawal from the course, or earning a grade the student deemed insufficient, and then completing the course. Repeaters that did not complete were not counted as completer-repeaters.

The Fall 2017 sections of MAT 152 served as the control groups and the Spring 2018 sections were the treatment groups (Table 2). All comprehensive gradebooks are stored on MyLabsPlus. MyLabsPlus is a web-based digital content platform. The server is maintained by Pearson. To access the server, you must login via a password protected account that is created by the coordinators of the mathematics department at MCC and be an actively enrolled student in the course or the instructor for the course. Students can only access a view-only report of their personal grades and are unable to update or change grades or see other student grades.

Table 1

*Course Demographics*

| Semester | Fall 2017 | Spring 2018 | Fall 2017 | Spring 2018 |
|---|---|---|---|---|
| Campus | Statesville | Statesville | Mooresville | Mooresville |
| Control/Treatment | Control | Treatment | Control | Treatment |
| # of students registered | 21 | 25 | 25 | 26 |
| # of students completed | 14 | 21 | 24 | 24 |
| % of Completers | 67% | 84% | 96% | 93% |
| # of Dual Enrolled (DE) completers | 4 | 3 | 9 | 8 |
| % of DE Completers | 29% | 14% | 38% | 33% |
| # of completer-repeaters | 1 | 2 | 0 | 3 |
| # of students who dropped during Drop/Add or were a No Show (no grade is assigned in these cases) | 4 | 1 | 0 | 0 |
| # of Withdraws | 3 | 3 | 1 | 2 |
| #Male:#Female | 5:9 | 7:14 | 8:16 | 10:14 |
| #White:#NonWhite | 9:5 | 17:4 | 23:1 | 24:1 |

*Note:* Fall 2017 Statesville Control had a larger than usually number of No Shows.

There were two pretests and two posttests administered to all sections included in the study on the first day and second to last day of each course to measure differences between the two groups. Both sets of pretests and posttests were completed using a machine-readable paper on which students mark answers to questions (i.e., Scantron). These forms were stored in a locked file cabinet in the researcher's locked office. Tests were not graded until all four sections were completed in May 2018 and the tests were graded using a grading machine. One of the tests was the Comprehensive Assessment of Outcomes in a First Statistics (CAOS) course. It consists of 40 multiple choice questions. The CAOS test assesses students' statistical reasoning after any first course in statistics with a focus on statistical literacy and conceptual understanding (Regents of the University of Minnesota, 2006). It has a Cronbach alpha coefficient of .78.

The second test administered was the Constructivist Learning Environment Survey (CLES). The Constructivist Learning Environment Survey (CLES) is a 42-item questionnaire which uses a 5-point Likert scale measuring students' classroom experiences in six categories: Attitude, Personal Relevance, Critical Voice, Shared Control, Uncertainty, and Student Negotiation (CLES, 2017). The CLES questionnaire and a list of which questions measured each category can be found in Appendix C. Cronbach alpha scales in various studies for multiple versions of the CLES are consistently over .7 for each theme and for the overall survey (Johnson & McClure, 2004; Nix, Fraser, & Ledbetter, 2005; Ogbuehi &Fraser, 2007; Taylor, Fraser, & Fisher, 1997). For this study, only questions relating to Personal Relevance and Critical Voice were of interest. Personal Relevance questions measure a students' perceived relevance of the content learned in the classroom to their out-of-school experiences. These questions examine whether not students are able to read and write the world with mathematics by being able to recognize and apply the curriculum to real world situations (Gutstein, 2006). Critical Voice questions focus on empowerment, ultimately measuring the climate of the classroom and the extent to which students feel able to question or reflect on the teaching and learning process (Haney, Wang, Keil, & Zoffel, 2007; Lew, 2010; Taylor & Fraser, 1991). It is when students feel supported and valued that they share critical opinions allowing for learning and dialogue to take place. Both question categories measure key elements of critical pedagogies.

In all four courses, Triola's (2015) *Essentials of Statistics* ($5^{th}$ edition) served as the course text and as a source of course material and resources. Chapters 1 through 8 and section

10.2 and 10.3 served as the curriculum for all sections. Sections 5.4 and 6.6 were excluded from the curriculum. MyLabsPlus (MLP) was the courses' online homework platform, one homework per section, with all four courses being assigned the same questions (numerical answers may vary). Students had multiple, unlimited attempts on each question and could complete late homework for a 25% penalty. Homework is graded automatically by MLP. Grades are stored on the MLP web-based server via Pearson.

Table 2

*Section Groups*

| Group | Pretests | Intervention | Posttest |
|---|---|---|---|
| Mooresville Spring 2018 | O | X | O |
| Mooresville Fall 2017 | O | | O |
| Statesville Spring 2018 | O | X | O |
| Statesville Fall 2017 | O | | O |

Final exams and unit tests were created from the textbook's testbank for TestGen with all unit exams and the final exam remaining the same for each section. There were three unit exams, with the third unit exam being a take-home exam. All exams were administered on paper. The first two unit exams, chapters 1 to 3 and chapters 4 to 6, consisted of multiple-choice and short answer questions; the last unit test for chapters 7 and 8 was comprised of only short answer questions. Sections 10.2 and 10.3 were assessed on the final exam, which consisted of all multiple-choice questions and required the student to complete a machine-readable paper. All unit exams had a bonus, extra-credit question. The day before each unit exam, and two days before the final exam, served as a review period. Review tests were the same for both the control and treatment groups. Missed unit exams needed to be made up the next class period. There was no make-up session for missed final exams. Unit exams were graded by the researcher and were returned to students the next class period. Final exam papers were graded the day after the exam using a grading machine and were stored in a locked cabinet in the researcher's office. As per college policy, final exams were not returned as all final exams must be retained by the instructor for three years.

There were 10 labs assigned in all sections. There was one lab for chapters 1, 5, and 6, a single, combined lab for chapters 2 and 3, and two labs each for chapters 4, 7, and 8. There was no lab for sections 10.2 and 10.3. All labs were researcher-created, required the use of technology, and were completed in the classroom. Control groups labs consisted of hypothetical scenarios that students examined with the use of online and textbook resources and their calculator. Results were submitted either via an Excel or Word document or Quiz link in Moodle. Examples of labs for the control group can be found in Appendix A. Students had the option of completing the lab outside of the classroom either due to absence, preference, or through requiring additional time. Due dates for labs were one week after the scheduled class period. Late labs were accepted one week past the due date for a 25% penalty. Labs were graded by the researcher via Moodle or on the submitted hard copy and returned to the student. Copies of paper submissions were retained and stored in a locked cabinet in the researcher's locked office. Digital copies of labs exist both on the Moodle server and are saved on an encrypted external hard drive.

The treatment for the study was the implementation of the TSfSJ pedagogy in the Spring 2018 MAT 152 courses. The treatment consisted of embedding an overarching focus on issues of social justice relating to race, class, gender, and sexual orientation. This was achieved by adding examples, questions, and data exploring these topics to the PowerPoints and labs used with the control group. Labs were similar in design and theme, but incorporated real-world, local data to be explored in addition to, or in replace of, the fictional data used in the control. Datasets that were infused included, but were not limited to, salaries of Fortune 100 CEOs, traffic stop data for Iredell County, poverty and food insecurity for Iredell County and North Carolina overall, and topics related to health, race, and sexual identity from the Behavioral Risk Factor Survey for North Carolina. Sources for the data included government databases, research facilities, peer-review journal articles and studies, and reports from organizations deemed unbiased/neutral (such as NPR). Media reports were used sparingly and needed to cite reliable data sources (i.e., legitimate databases, peer-review studies, etc.). Submission of the labs continued to use Excel or Word documents or Quiz links in Moodle. Examples of labs for the treatment group can be found in Appendix B.

**Data Analysis**

Data for all graded assignments and course grades was downloaded from MLP and saved in an encrypted Excel file and stored on an encrypted external hard drive. The data for the pretest/posttest scores for the CAOS, noting if the student was repeating the course, and specifying the students' sex, was added to the same file (File 1). Each courses' aggregate responses to the pretest/posttest CLES for each of the questions for Personal Relevance and Critical Voice categories were entered by the researcher and saved in an encrypted Excel file (File 2) and stored on the encrypted external hard drive. For all Excel files, each course was organized on a separate spreadsheet. Data for participants who completed both sets of pretests and posttests was entered in the software Statistical Package for the Social Sciences 24 (SPSS). A total of 12 participants were excluded due to not completing both sets of the pretests and posttests. From Fall 2017, four participants were eliminated from the Statesville control course and five from the Mooresville control course. There were two participants eliminated from the

Spring 2018 Statesville treatment course and one participant from the Spring 2018 Mooresville treatment course. Final exam scores and overall over course grades for all participants who completed the course was also entered into SPSS.

A variety of descriptive and inferential statistics were used to analyze the data. The type of analysis used was determined by the level of measurement of data collected. Differences between pretest and posttest CAOS scores were analyzed using a Paired *t*-test or a Mann-Whitney U test, depending on the distribution of the data. A Mann-Whitney U test was used to compare CLES pretest and posttest results for questions relating to Critical Voice and Personal Relevance as the collected data was ordinal. Finally, to examine course success between groups, a two-population proportion z-test was used to measure the impact of the treatment on persistence and successful completion.

**Trustworthiness of the Analysis**

While quasi-experimental designs are often recommended for educational evaluations, they are less desirable than a true experiment as they often fail to produce the same results (Steiner et al., 2009). In this section, we will explore the challenges to credibility of a NCGD and examine the reliability and validity of the CAOS and CLES instruments.

**Nonequivalent Control Group Designs.** While NCGD is one of the most popular quasi-experimental designs, especially in educational settings, there are notable threats to the trustworthiness of its findings. Christensen, Johnson, and Turner (2014) identify that NCGD participants are more likely to

> (1) drop out of one group than from another group, (2) mature at different rates in the different groups, (3) be differently assessed by the measurement process in different groups, (4) "regress-to-the-mean" at different rates in the different groups and (5) react differently to nontreatment-related events that occur between the pretest and posttest. (p. 274).

All five threats stem from the lack of randomization. As groups are not randomized, groups could differ systematically prior to the treatment being administered, creating a threat to both external and internal validity due to selection and maturation (Reichardt, 2009). Selection threat is when groups are formed due to differential (non-randomized) selection. When groups are not formed by random selection and assignment, then they may vary in nature due to extraneous variables, such as age or gender. These variables could create biases between the treatment and control groups, referred to as selection bias. Maturation threat is a result of the passage of time and its impact on both biological and psychological processes, such as learning, boredom, and hunger, and scores; it is not related to specific external events but to the individual (Campbell & Stanley, 1963; Christenson et al., 2014).

The use of a pretest and posttest in both treatment and control groups can help to minimize these threats. The pretest scores can help to identify differences between the intact groups being compared, if differences exist. This allows a researcher to control for preexisting differences when comparing results on the posttest between the treatment and control groups (Morris, 2008). Specifically, pretests help to identify and possibly control the threat of selection bias by using change scores (Christenson et al., 2014). Change scores are identified by finding the difference between the pretest and posttest for each participant, helping to isolate the effect of the treatment minimizing an external threat of selection. However, selection could still impact maturation as different groups mature at different rates, making internal validity a concern with NCGD pretest/posttest designs in educational settings (Reichardt, 2009; Steiner, Wroblewski, & Cook, 2009).

Campbell and Stanley (1963) warn that “selection-maturation interaction (or a selection-history interaction or a selection-testing interaction) could be mistaken for the effect of [the treatment], and thus represents a threat to the internal validity of the experiment" (p. 48). This threat to internal validity makes analyzing data from NCGDs challenging. The two most common statistical approaches are to use a Paired *t*-test to test the mean difference of the change scores and an analysis of covariance (ANCOVA) using change scores of adjusted (also referred to as partialled) initial scores (Morris, 2008; Steiner et al., 2009; Wright, 2006). As assignment to a group in a NCGD is not randomized, we have a comparison group, not a control group, and selection-maturation differences are not properly accounted for (Muijs, 2004; Reichardt, 2009). The use of an ANCOVA is often the preferred method as it considers effects of selection differences by statistically adjusting the posttest scores for any pretest differences found between two groups. However, a Paired *t*-test was implemented as the use of a pretest served as a way to confirm that the groups were similar, allowing the NCGD to approach an experimental design (Campbell & Stanley, 1966; Christenson et al., 2014; Reichardt, 2009).

**CAOS.** Two sets of pretest/posttest exams were administered during the study. The Comprehensive Assessment of Outcomes in Statistics (CAOS) was one of these tests. The CAOS is a 40-item multiple choice tests designed to measure students' understanding of concepts that are commonly found in an introductory statistics course (Hahs-Vaugh, Acquaye, Griffith, Jo, Matthews, & Achraya, 2017). Over three years, the test was developed through the Assessment Resource Tools for Improving Statistical Thinking (ARTIST) project funded by The National Science Foundation (NSF). The CAOS underwent four revisions through increasing larger and more varied test groups before being finalized.

Original test items were acquired from post-secondary instructors and leaders in statistics education or written by test developers and sent for feedback from advisors and testers. The first version of the CAOS had 34 test questions and was piloted with a group of introductory statistics students in 2004. Data received from the pilot study was used to create a second version of the CAOS. The second version, consisting of 37 test questions, was used in the first large scale class testing of the online instrument in early 2005 (DelMas, Garfield, Ooms, & Chance, 2007). Results from the nearly 1000 participants were used to create the third version of the CAOS that was tested with 30 faculty graders of the Advanced Placement (AP) Statistics exam in summer 2005. It was the feedback from this group that created the fourth and final version of the CAOS.

After the content was rated by a group of 18 members of the advisory and editorial boards of the Consortium for the Advancement of Undergraduate Statistics Education (CAUSE), the finalized version of the CAOS was administered online and on paper for analysis. Nearly 1500 introductory statistics students, taught by 35 instructors from 33 higher education institutions from 21 states across the United States, took the CAOS in a classroom or controlled setting. Students enrolled in AP courses were not included. An analysis of internal consistency of the 40 items on the CAOS produced a Cronbach's alpha coefficient of 0.82, displaying adequate internal consistency by a variety of scales (DelMas et al., 2007; Hahs-Vaugh et al., 2017; Sabbag & Zieffler, 2015). "The CAOS test was judged to have acceptable internal consistency for students enrolled in college-level, nonmathematical introductory statistics courses given that the estimated internal consistency reliability is well above the range of suggested lower limits [of 0.5 to 0.7]" (DelMas et al., 2007, p. 33).

**CLES.** The second pretest/posttest was the Constructivist Learning Environment Survey (CLES). Since originally being presented to the American Educational Research Association in 1991, the CLES has been validated in a variety of studies in numerous countries and exists in several versions (Nix, Fraser, & Ledbetter, 2005; Ogbehi & Fraser, 2007 ; Vennix, Brok, & Taconis, 2017). It was developed to help researchers and instructors assess if a classroom environment is consistent with a constructivist approach and to reflect on their practice (Taylor & Fraser, 1991). The original CLES, consisting of 58 Likert items assessing five scales (autonomy, prior knowledge, negotiation, attitude, and student-centeredness), was field tested in 12 Australian secondary schools by a total of 508 students from eighth grade to twelfth grade in 26 different science and mathematics classes.

Analysis of the results led to the refinement of the scales and the removal of 30 items (Taylor & Fraser, 1991). The refined CLES was found to have satisfactory internal consistency reliability, producing Cronbach's alpha coefficients between 0.69 and 0.85 for each scale, as well as discriminant validity and factorial validity (Taylor & Fraser, 1991). Various recent versions of the CLES with American students and teachers have since shown higher Cronbach's alpha coefficients ranging from 0.72 to as high as 0.94 (Johnson & McClure, 2003).

The version of the CLES used in this study was a 1994 revision that was developed using a critical constructivist theoretical framework. This version of the CLES measures the extent to which a classroom environment emphasizes: (a) making science/mathematics relevant to the world outside of school; (b) engaging students in reflective dialogue; (c) inviting students to share control of instructional design, management, and evaluation of their learning; (d) empowering students to express concern about the quality of learning activities; and (e) creating a learning experience that is uncertain in nature of scientific and mathematical knowledge for the student (Taylor, Fraser, & White, 1994). It is comprised of 42 items using a five-point Likert scale. There are five scales relating to critical theory, personal relevance, shared control, critical voice, student negotiation, and uncertainty, and an attitude scale, which measure satisfaction of the class overall. It was trialed using an interpretive research framework in an eighth-grade mathematics class in a public, metropolitan school.

The 34 responses found Cronbach's alpha reliability scales ranging from 0.54 to 0.85 with low alphas largely attributed to negatively worded items (Taylor, Fraser, & White, 1994). Correlations between the attitude scale and the five scales relating to critical theory ranged from 0.26 to 0.55 (significance at $|r| > 0.339$), with some intercorrelations that were also significant. Other versions of the CLES share similar results (Nix et al., 2005; Ogbuehi & Fraser, 2007; Vennix et al., 2017). For this study, only items relating to Personal Relevance ($\alpha = 0.81$, $r = 0.55$) and Critical Voice ($\alpha = 0.79$, $r = 0.33$) were of significance.

**Overall Limitations.** The most significant limitation of the study was a lack of randomization. However, due to the educational setting, it was not feasible to design a true experiment. Additionally, while the pretest is recommended for measuring preexisting differences unrelated to the treatment, the existence of a pretest in a control group setting can produce gains in performance without treatment as it provides practice answering test questions (Cook et al., 2002). Last, treatment and control groups, while measured the same academic year, were measured in two different semesters. This can compound maturation threats but also, due to weather related closings and delays, impact the effectiveness of the treatment.

# Chapter Four: Results

In this chapter, the results of the study are reviewed. The following research questions guided the study (1) does implementing a critical pedagogy increase student success? Student success is defined as completing the course with a C or higher and, (2) does use of a critical pedagogy increase the student's sense of mathematical empowerment? The CAOS pre- and posttests, final exam grades, course grades, and course completion rates were used to measure student success. MP was measured using a subset of questions from the CLES. Data sources, analysis methods, and variables are listed for each research question in Table 3.

## Research Question One

Several analyses were run to examine the impact of the treatment on student success. While student success is defined in this study as completing the course with a course grade of 70 or higher, pretest/posttest CAOS scores, persistence, and final exam scores were also analyzed.

**Course Grades.** A Kolmogorov-Smirnov Test (K-S Test) was run on each section's final course grades. For the Statesville sections, both the control ($D(10) = .186$, $p > .05$) and treatment ($D(19) = .152$, $p > .05$) were deemed to be individually normally distributed by the K-S. A Levene's Test for Equality of Variances showed that the two sections had equal variances. An independent *t*-test did not indicate a statistical difference between the treatment ($M = 83.1$, $SE = 2.94$) and control ($M = 85.61$ $SE = 3.53$), $t(27)= 0.523$, $p > .05$.

The Mooresville control ($D(19) = .168$, $p > .05$) and treatment ($D(23) = .168$, $p > .05$) sections were also both normally distributed. A Levene's Test for Equality of Variances showed that the two sections did not have equal variances. An independent *t*-test did indicate a statistical difference between the treatment ($M = 91.39$, $SE = 1.69$) and control ($M = 80.65$ $SE = 3.82$), $t(24.922)= -2.573$, $p < .05$.

Table 3

*Data Sources and Analysis for Research Questions*

| Location | Research Question | Data Source | Data Analysis Method |
|---|---|---|---|
| Statesville Control – Statesville Treatment | Does implementing a critical pedagogy increase student success? | 1. Course Grades<br>2. Final Exam Grades<br>3. CAOS pre/post test<br>4. Course Completion Rates | 1. Independent *t*-test<br>2. Independent *t*-test<br>3. Paired *t*-test<br>4. Two population proportion *z*-test |
| Statesville Control – Statesville Treatment | Does use of a critical pedagogy increase the student's sense of mathematical empowerment? | CLES pre-post survey - Personal Relevance (PV) and Critical Voice (CV) questions | Mann-Whitney U Test |
| Mooresville Control - Mooresville Treatment | Does implementing a critical pedagogy increase student success? | 1. Course Grades<br>2. Final Exam Grades<br>3. CAOS pre/post test<br>4. Course Completion Rates | 1. Independent *t*-test<br>2. Mann-Whitney U Test<br>3. Paired *t*-test and Related-Samples Wilcoxon signed rank Test<br>4. Two population proportion *z*-test |
| Mooresville Control - Mooresville Treatment | Does use of a critical pedagogy increase the student's sense of mathematical empowerment? | CLES pre-post survey - Personal Relevance (PV) and Critical Voice (CV) questions | Mann-Whitney U Test |
| Pooled control – Pooled treatment | Does implementing a critical pedagogy increase student success? | Course Completion Rates | Two population proportion *z*-test |
| Pooled control – Pooled treatment | Does use of a critical pedagogy increase the student's sense of mathematical empowerment? | CLES pre-post survey - Personal Relevance (PV) and Critical Voice (CV) questions | Two population proportion *z*-test |

**Final Exam Grades.** A K-S Test was run on each section's final exam grades (max score of 40). Both Statesville sections were deemed to be individually normally distributed for the control ($D(10) = .186$, $p > .05$) and for the treatment ($D(19) = .154$, $p > .05$) by the K-S. A Levene's Test for Equality of Variances showed that the two sections had equal variances. The independent *t*-test did not find a statistical difference between the treatment ($M = 31.21$, $SE = 1.37$) and control ($M = 32.8$, $SE = 1.52$), $t(27) = .726$, $p > .05$.

The K-S test for the Mooresville control ($D(19) = .204$, $p < .05$) and treatment ($D(23) = .190$, $p < .05$) were not found to be normally distributed. Due to the lack of normality, an independent Mann-Whitney U Test was used to compare the final course grades of the sections and found there was a statistically significant difference between the control ($Mdn = 28$) and treatment ($Mdn = 35$) sections, $U = 358.50$, $z = 3.55$.

**CAOS pretest/posttest scores.** An independent *t*-test showed that control groups were not statistically different from their treatment groups. Levene's Test of Equality of Variance was not significant for the Statesville control and treatment pretest CAOS scores ($F(1, 27) = 1.173$, $p > .05$) showing the variances of the pretests for the control and treatment to be equal. Independent *t*-tests of the pretest CAOS scores confirmed that the Statesville control and treatment groups did not differ statistically, $t(27) = -0.384$, $p > .05$. The same procedure for the Mooresville control and treatment was implemented. Levene's Test of Equality of Variance was not significant for the Mooresville control and treatment pretest CAOS scores ($F(1, 40) = 1.774$, $p > .05$). Independent *t*-tests of the pretest CAOS scores confirmed that the Mooresville control and treatment groups did not differ statistically, $t(40) = -1.541$, $p > .05$. Table 4 shows the SPSS coding for each treatment and control group.

Last, an analysis of variance (ANOVA) was used to examine all four groups. An Independent sample Kruskal-Wallis test verified all four groups had the same distribution ($H(3) = 1.528$, $p > .05$). Levene's test of homogeneity of variances indicated that the assumption of homogeneity of variance had not been violated, $F(3, 67) = 1.243$, $p > .05$. The ANOVA found all groups to have equal means with group association having minimal impact of pretest scores ($F(3, 67) = 1.063$, $p > .05$, $r = .21$).

Table 4

*SPSS Coding for CAOS Scores*

| Variable | Coding |
|---|---|
| SV CAOS Groups | svf = Statesville control<br>svs = Statesville treatment |
| MV CAOS Groups | mvf = Mooresville control<br>mvs = Mooresville treatment |
| SVPreCAOS | Pretest scores for all Statesville sections |
| SVPostCAOS | Posttest scores for all Statesville sections |
| MVPreCAOS | Pretest scores for all Mooresville sections |
| MVPostCAOS | Posttest score for all Mooresville sections |
| AllPreCAOS | Pretest scores for all sections |

***Statesville sections.*** Shapiro-Wilk (S-W) tests showed both the pretest ($W(10) = .910$, $p > 0.05$) and posttest ($W(10) = .910$, $p > 0.05$) were normally distributed for the control group. The treatment group pretests ($W(19) = .949$, $p > 0.05$) and posttests ($W(19) = .934$, $p > 0.05$) were also found to be normally distributed. Normal Q-Q plots and boxplots confirmed that there were no outliers. A linear relationship between the pretest and posttest of the CAOS was verified with a matrix scatter for both the treatment and control. Levene's Test of Equality of Error Variance was not significant ($F(1, 27) = .033$, $p > .05$) showing the variances of the pretests for the control and treatment to be equal. Independent *t*-tests were used to explore the data. Neither the control,

$t(9) = -0.9.19$, $p > .05$, nor the treatment, $t(18) = -.109$, $p > .05$, showed a significant difference between the pretest and posttest. Descriptive statistics for all sections can be found in Table 5.

Table 5

*Descriptive Statistics for the CAOS pretests and posttests*

| Section | Test | Mean | Standard Error |
|---|---|---|---|
| Statesville Control | Pretest | 15.80 | 1.42 |
| Statesville Control | Posttest | 17.00 | 1.69 |
| Statesville Treatment | Pretest | 15.26 | 0.69 |
| Statesville Treatment | Posttest | 15.37 | 0.82 |
| Mooresville Control | Pretest | 15.00 | .78 |
| Mooresville Control | Posttest | 15.32 | .70 |
| Mooresville Treatment | Pretest | 16.96 | .96 |
| Mooresville Treatment | Posttest | 18.57 | 1.03 |

***Mooresville sections.*** Shapiro-Wilk (S-W) tests showed both the pretest ($W(19) = .929$, $p > 0.05$) and posttest ($W(19) = .965$, $p > 0.05$) were normally distributed for the Mooresville control group. However, S-W tests showed the treatment was not normally distributed for the pretest ($W(23) = .900$ $p < 0.05$) or posttest ($W(23) = .910$, $p < 0.05$). The Normal Q-Q plot and boxplot confirmed there were two outliers in the treatment (scores 7 and 33). When the outliers were removed from the data set, Shapiro-Wilk reported the treatment as normally distributed, $W(21) = .918$, $p > 0.05$. A linear relationship between pretest and posttest of the CAOS scores was verified with a matrix scatter for both the treatment and control. Levene's Test of Equality of Error Variance was not significant showing the variances of the pretests for the control and treatment to be equal, $F(1, 38) = 0.805$ , $p > .05$. The data was retested with outliers remaining in the set. A linear relationship between pretest and posttest of the CAOS was verified with a matrix scatter for both the treatment and control. Levene's Test of Equality of Error Variance showed the variances of the two groups to be equal as the test was not found be significant ($F$(1, 40) = 3.196, $p > 0.05$).

A paired $t$-test was used to investigate the control and a Related-Samples Wilcoxon signed rank test was used for the treatment, as the treatment was originally found not to be normally distributed. The paired $t$-test did not find a statistically significant difference between the pretest and posttest for the control, $t(18) = -0.637$, $p < .05$. However, the difference between the pretest (*Mdn* = 16) and the posttest (*Mdn* = 18) for the treatment was significant, $T = 188.00$, $p < .05$. When outliers were removed from the treatment, a paired $t$-test also found a statistical difference in the treatment group, $t(20) = -2.274$, $p < .05$.

Table 6

*Distribution of Completion/Withdrawals and Successful/Unsuccessful*

| Section (SPSS code) | Complete (1) | Withdrawal (2) | Successful Completion (ABC) | Unsuccessful /incomplete (DFW) |
|---|---|---|---|---|
| Statesville Control (svf) | 14 (82.4%) | 3 (17.6%) | 13(76.5%) | 4 (23.5%) |
| Statesville Treatment (svs) | 21 (87.5%) | 3 (12.5%) | 18 (75%) | 6 (25%) |
| Mooresville Control (mvf) | 21 (96%) | 1 (4%) | 19 (86%) | 3 (14%) |
| Mooresville Treatment (mvs) | 24 (92.3%) | 2 (7.7%) | 24 (92.3%) | 2 (7.7%) |

**Persistence.** A population proportion test was used to measure the effect of treatment on persistence. Tests were completed via TI-84 SE with z scores rounded to the hundredths. Assuming an alternate hypothesis of svfComplete < svsComplete, completion of the course (all scores but a W) was not found to be impacted by the treatment, $z = -0.4$, $p > .05$. The same structure was used to test mvfComplete < mvsComplete. Once again, completion was not impacted by the treatment, $z = -0.67$, $p > .05$. When pooled, controlCompletion < treatmentCompletion, the impact of the treatment remained insignificant, $z = -0.04$, $p > .05$. Successfully completing the course (a course letter grade of C or higher) was also not significant for both svfABC < svsABC ($z = 1.08$, $p > .05$) or mvf ABC < mvs ($z = 1.08$, $p > .05$). The data was again pooled to measure the impact of the treatment (controlABC < treatmentABC) and found not to be significant, $z = -0.94$, $p > .05$. Table 6 provides completion/withdrawal and successful/unsuccessful completion rates.

**Research Question Two**

To measure the effect of the treatment on MP, the CLES was given as a presurvey and postsurvey in both the treatment and the control. The CLES is a 5-point Likert Survey with ordinal rankings of Almost Always, Often, Sometimes, Seldom, and Almost Never. While the CLES measures six scales, only two were of importance in this study: personal relevance and critical voice. Both the personal relevance (PR) and critical voice (CV) scales consisted of seven questions. Of the seven, two are reversed scored in the PR scale and one in the CV scale. Reliability analyses was run on the questions, individually and combined, both with and without the reversed scored questions. Only the PR scale with the reversed scored items showed a lack of internal consistency. The PR scale had a Cronbach Alpha of 0.970 without reverse scored items and $\alpha = 0.612$ with. Reversed scored PR items alone had an $\alpha = 0.754$. The CV scale without the reversed score ($\alpha = 0.970$) and with ($\alpha = 0.880$) both showed internal consistency. Overall, items in both scales had an $\alpha = 0.983$ with reversed scored items and $\alpha = 0.906$ without.

While the study did not explore the relationship between PR and CV, inter-item correlations were measured. An inter-item correlation matrix between non-reversed in the PR scale items had correlations ranging from 0.790 to 0.926 (excluding self-correlations). Reversed PR items had an $r = 0.605$. For non-reversed CV items, correlations ranged from 0.781 to 0.907 (excluding self-correlations). A factor analysis of non-reversed PR and CV items showed correlations ranging from 0.762 to 0.949. Bartlett's Test of Sphericity confirmed the relationship between the items ($\chi^2$ (55) = 2485, $p < 0.001$) and the Kaiser-Meyer-Olkin measure of sampling adequacy was measured at 0.949.

Table 7 provides the mean and standard error of each question of the CLES by test and section. Due to the ordinal nature of the data a Mann-Whitney U (MW U) test was administered on each question per section. While the surveys are noted as 'pre' and 'post', they should not be considered paired data sets. The presurvey asked participants to rate their experiences in past mathematics/statistics courses and the postsurvey asked them to rate the control or treatment class. All reported test data for all questions can be found in Tables 8 through 11. Questions PR13, PR30, CV15, CV27, and CV39 were found to have a significant difference between the mean ranks for the Statesville control. However, all questions showed significance in the treatment.

For the Mooresville sections, nearly all questions showed a significant difference between the mean ranks for both the control and the treatment (Table 10). The Mooresville control only lacked significance for PR1, PR13, and PR30. All CV questions were significant. The Mooresville treatment gained significance in the missing PR questions from the control, but lost significance in CV3 and CV15. Overall, the Statesville treatment showed significant growth in PR and CV while the Mooresville treatment gained PR while losing CV (Tables 8 and 9).

Table 7
*Mean (Standard Error) for CLES Questions by Section*

| | Statesville Control (N = 9) | | Mooresville Control (N = 19) | | Statesville Treatment (N = 19) | | Mooresville Treatment (N = 23) | |
|---|---|---|---|---|---|---|---|---|
| Question | Pretest | Posttest | Pretest | Posttest | Pretest | Posttest | Pretest | Posttest |
| PR1 | 3.67 (0.41) | 3.56 (0.29) | 2.74 (0.26) | 3.32 (0.22) | 2.57 (0.27) | 4.32 (0.24) | 3.17 (0.24) | 4.48 (0.15) |
| CV3 | 3.33 (0.55) | 4.33 (0.33) | 2.58 (0.30) | 3.89 (0.30) | 2.95 (0.33) | 4.21 (0.26) | 3.09 (0.29) | 3.52 (0.33) |
| PR7 | 2.89 (0.48) | 3.56 (.018) | 2.26 (0.25) | 3.32 (0.22) | 2.37 (0.26) | 4.42 (0.25) | 2.57 (0.22) | 4.26 (0.17) |
| CV9 | 3.11 (0.45) | 4.11 (0.26) | 2.26 (0.25) | 3.32 (0.22) | 2.63 (0.24) | 4.37 (0.26) | 2.91 (0.23) | 3.70 (0.28) |
| PR13 | 3.78 (0.40) | 3.89 (0.26) | 3.32 (0.25) | 3.68 (0.27) | 3.37 (0.27) | 4.32 (0.25) | 3.61 (0.22) | 4.17 (0.20) |
| CV15 | 2.22 (0.28) | 3.67 (0.29) | 2.42 (0.21) | 3.68 (0.27) | 2.37 (0.31) | 3.58 (0.28) | 2.87 (0.28) | 3.30 (0.27) |
| PR19 | 3.22 (0.40) | 3.78 (0.36) | 2.42 (0.26) | 3.42 (0.28) | 2.42 (0.28) | 3.95 (0.29) | 2.70 (0.24) | 4.09 (0.23) |
| CV21 | 2.22 (0.36) | 3.22 (0.36) | 2.68 (0.19) | 3.95 (0.19) | 2.58 (0.31) | 3.68 (0.31) | 2.65 (0.26) | 3.48 (0.24) |
| PR25 | 3.11 (0.45) | 3.78 (0.32) | 2.47 (0.18) | 3.26 (0.26) | 2.26 (0.23) | 3.95 (0.31) | 2.83 (0.22) | 4.00 (0.24) |
| CV27 | 3.44 (0.38) | 5.00 (0) | 3.37 (0.24) | 4.32 (0.22) | 3.63 (0.31) | 4.53 (0.22) | 3.78 (0.27) | 4.48 (0.20) |
| PR30* | 2.78 (0.22) | 3.78 (0.22) | 2.47 (0.25) | 3.00 0.29) | 2.89 (0.29) | 1.79 (0.16) | 3.04 (0.20) | 4.17 (0.16) |
| CV32 | 3.89 (0.42) | 4.22 (0.36) | 3.37 (0.27) | 4.21 (0.26) | 3.21 (0.32) | 4.32 (0.25) | 3.48 (0.26) | 4.61 (0.12) |
| PR37* | 3.44 (0.29) | 4.00 (0.29) | 2.74 (0.23) | 3.58 (0.28) | 3.21 (0.26) | 4.37 (0.22) | 2.96 (0.26) | 4.43 (0.14) |
| CV39* | 2.78 (0.40) | 4.22 (0.46) | 2.89 (0.21) | 4.47 (0.23) | 3.47 (0.25) | 4.42 (0.19) | 3.30 (0.25) | 4.00 (0.19) |

*Note:* *Reverse Scoring

Table 8
*Statesville Mann-Whitney U Results*

| | Statesville Control | | | Statesville Treatment | | |
|---|---|---|---|---|---|---|
| Question | *U* | *z* | *p* | *U* | *z* | *p* |
| PR1 | 38 | -0.234 | 0.815 | 44 | -4.095 | .000* |
| PR7 | 26.5 | -1.281 | 0.200 | 38.5 | -4.299 | .000* |
| PR13 | 40.5 | 0 | 1.000 | 91.5 | -2.728 | .006* |
| PR19 | 29 | -1.054 | 0.292 | 67.5 | -3.385 | .001* |
| PR25 | 28 | -1.17 | 0.242 | 59 | -3.622 | .000* |
| PR30** | 13.0 | -2.599 | 0.009* | 82.5 | -2.985 | 0. 003* |
| PR37** | 27.0 | -1.260 | 0.208 | 78.5 | -3.101 | 0.002* |
| CV3 | 26 | -1.357 | 0.175 | 90 | -2.761 | 0.006* |
| CV9 | 22 | -1.688 | 0.091 | 45 | -4.103 | 0* |
| CV15 | 10 | -2.889 | 0.004* | 92 | -2.642 | 0.008* |
| CV21 | 20 | -1.878 | 0.060 | 101 | -2.376 | 0.018* |
| CV27 | 9 | -3.177 | 0.001* | 95 | -2.703 | 0.007* |
| CV32 | 34.5 | -0.571 | 0.568 | 87 | -2.866 | 0.004* |
| CV39** | 14.0 | -2.415 | 0.016* | 85.5 | -2.917 | 0.004* |

Note: * $p < .05$, ** reverse scoring

Table 9

*Mooresville Mann-Whitney U Results*

| | Mooresville Control | | | Mooresville Treatment | | |
|---|---|---|---|---|---|---|
| Question | *U* | *z* | *p* | *U* | *z* | *p* |
| PR1 | 127 | -1.640 | 0.101 | 96.000 | -3.864 | 0.000* |
| PR7 | 80.5 | -3.041 | 0.002* | 60.500 | -4.607 | 0.000* |
| PR13 | 143.5 | -1.118 | 0.263 | 179.000 | -2.019 | 0.044* |
| PR19 | 96.5 | -2.515 | 0.012* | 104.000 | -3.635 | 0.000* |
| PR25 | 103.5 | -2.386 | 0.017* | 118.000 | -3.317 | 0.001* |
| PR30** | 132.50 | -1.444 | 0.149 | 96.5 | -3.870 | 0. 000* |
| PR37** | 106.0 | -2.246 | 0.025* | 97.0 | -3.835 | 0.000* |
| CV3 | 87.5 | -2.802 | 0.005* | 212.500 | -1.172 | 0.241 |
| CV9 | 80.5 | -3.041 | 0.021* | 171.500 | -2.113 | 0.035* |
| CV15 | 71.5 | -3.271 | 0.001* | 211.000 | -1.212 | 0.225 |
| CV21 | 54 | -3.871 | 0.000* | 163.500 | -2.290 | 0.022* |
| CV27 | 91.50 | -2.720 | 0.007* | 178.500 | -2.063 | 0.039* |
| CV32 | 102.50 | -2.372 | 0.018* | 122.500 | -3.327 | 0.001* |
| CV39** | 51.0 | -3.962 | 0.000* | 172.50 | -2.095 | 0.036* |

Note: * p < .05, ** reverse scoring

Table 10
*Statesville Mann-Whitney U Data*

| | Statesville Control (n = 9) | | | | Statesville Treatment (n = 19) | | | |
|---|---|---|---|---|---|---|---|---|
| Question | Pre/Post | Mdn | Mean Rank | Sum of Ranks | Pre/Post | Mdn | Mean Rank | Sum of Ranks |
| PR1 | Pre | 4 | 9.78 | 88.00 | Pre | 3 | 12.32 | 234.00 |
| | Post | 3 | 9.22 | 83.00 | Post | 5 | 26.68 | 507.00 |
| PR7 | Pre | 2 | 7.94 | 71.50 | Pre | 2 | 12.03 | 228.50 |
| | Post | 4 | 11.06 | 99.50 | Post | 5 | 26.97 | 512.50 |
| PR13 | Pre | 4 | 9.50 | 85.50 | Pre | 3 | 14.82 | 281.50 |
| | Post | 4 | 9.50 | 85.50 | Post | 5 | 24.18 | 459.50 |
| PR19 | Pre | 3 | 8.22 | 74.00 | Pre | 3 | 13.55 | 257.50 |
| | Post | 4 | 10.78 | 97.00 | Post | 4 | 25.45 | 483.50 |
| PR25 | Pre | 3 | 8.11 | 73.00 | Pre | 2 | 13.11 | 249.00 |
| | Post | 3 | 10.89 | 98.00 | Post | 4 | 25.89 | 492.00 |
| PR30 | Pre | 3 | 6.44 | 58.0 | Pre | 3 | 24.66 | 468.50 |
| | Post | 4 | 12.56 | 113.00 | Post | 2 | 14.34 | 272.50 |
| PR37 | Pre | 3 | 8.00 | 72.00 | Pre | 3 | 14.13 | 268.50 |
| | Post | 4 | 11.00 | 99.00 | Post | 5 | 24.87 | 472.50 |
| CV3 | Pre | 4 | 7.89 | 71.00 | Pre | 3 | 14.74 | 280.00 |
| | Post | 5 | 11.11 | 100.00 | Post | 5 | 24.26 | 461.00 |
| CV9 | Pre | 3 | 7.44 | 67.00 | Pre | 3 | 12.37 | 235.00 |
| | Post | 4 | 11.56 | 104.00 | Post | 5 | 26.63 | 506.00 |
| CV15 | Pre | 2 | 6.11 | 55.00 | Pre | 2 | 14.84 | 282.00 |
| | Post | 3 | 12.89 | 116.00 | Post | 4 | 24.16 | 459.00 |
| CV21 | Pre | 2 | 7.22 | 65.00 | Pre | 3 | 15.32 | 291.00 |
| | Post | 4 | 11.78 | 106.00 | Post | 4 | 23.68 | 450.00 |
| CV27 | Pre | 3 | 6.00 | 54.00 | Pre | 4 | 15.00 | 285.00 |
| | Post | 5 | 13.00 | 117.00 | Post | 5 | 24.00 | 456.00 |
| CV32 | Pre | 4 | 8.83 | 79.50 | Pre | 4 | 14.58 | 277.00 |
| | Post | 5 | 10.17 | 91.50 | Post | 5 | 24.42 | 464.00 |
| CV39 | Pre | 3 | 6.56 | 59.00 | Pre | 4 | 14.50 | 275.50 |
| | Post | 5 | 12.44 | 112.00 | Post | 5 | 24.50 | 465.50 |

Table 11
*Mooresville Mann-Whitney U Data*

| | Mooresville Control (n = 19) | | | | Mooresville Treatment (n = 23) | | | |
|---|---|---|---|---|---|---|---|---|
| Question | Pre/Post | Mdn | Mean Rank | Sum of Ranks | Pre/Post | Mdn | Mean Rank | Sum of Ranks |
| PR1 | Pre | 3 | 16.68 | 317.00 | Pre | 3 | 16.17 | 372.00 |
| | Post | 3 | 22.32 | 424.00 | Post | 5 | 30.83 | 709.00 |
| PR7 | Pre | 2 | 14.24 | 270.50 | Pre | 2 | 14.63 | 336.50 |
| | Post | 3 | 24.76 | 470.50 | Post | 4 | 32.37 | 744.50 |
| PR13 | Pre | 3 | 17.55 | 333.50 | Pre | 4 | 19.78 | 455.00 |
| | Post | 4 | 21.45 | 407.50 | Post | 4 | 27.22 | 626.00 |
| PR19 | Pre | 2 | 15.08 | 286.50 | Pre | 3 | 16.52 | 380.00 |
| | Post | 4 | 23.92 | 454.50 | Post | 5 | 30.48 | 701.00 |
| PR25 | Pre | 2 | 15.45 | 293.50 | Pre | 3 | 17.13 | 394.00 |
| | Post | 3 | 23.55 | 447.50 | Post | 4 | 29.87 | 687.00 |
| PR30 | Pre | 2 | 16.97 | 322.50 | Pre | 3 | 16.20 | 372.50 |
| | Post | 3 | 22.03 | 418.50 | Post | 4 | 30.80 | 708.50 |
| PR37 | Pre | 3 | 15.58 | 296.00 | Pre | 2 | 16.22 | 373.00 |
| | Post | 4 | 23.42 | 445.00 | Post | 5 | 30.78 | 708.00 |
| CV3 | Pre | 2 | 14.61 | 277.50 | Pre | 3 | 21.24 | 488.50 |
| | Post | 4 | 24.39 | 463.50 | Post | 4 | 25.76 | 592.50 |
| CV9 | Pre | 2 | 14.24 | 270.50 | Pre | 3 | 19.46 | 447.50 |
| | Post | 3 | 24.76 | 470.50 | Post | 4 | 27.54 | 633.50 |
| CV15 | Pre | 2 | 13.76 | 261.50 | Pre | 3 | 21.17 | 487.00 |
| | Post | 4 | 25.24 | 479.50 | Post | 3 | 25.83 | 594.00 |
| CV21 | Pre | 3 | 12.84 | 244.00 | Pre | 3 | 19.11 | 439.50 |
| | Post | 4 | 26.16 | 497.00 | Post | 4 | 27.89 | 641.50 |
| CV27 | Pre | 3 | 14.82 | 281.50 | Pre | 4 | 19.76 | 454.50 |
| | Post | 5 | 24.18 | 459.50 | Post | 5 | 27.24 | 626.50 |
| CV32 | Pre | 3 | 15.39 | 292.50 | Pre | 4 | 17.33 | 398.50 |
| | Post | 5 | 23.61 | 448.50 | Post | 5 | 29.67 | 682.50 |
| CV39 | Pre | 3 | 12.68 | 241.00 | Pre | 3 | 19.50 | 448.50 |
| | Post | 5 | 26.32 | 500.00 | Post | 4 | 27.50 | 632.50 |

Table 12

*Two Population Proportion Test for Control Groups – Positive Wording*

| | Number of AAO Reponses | | | | % of AAOs | | | |
|---|---|---|---|---|---|---|---|---|
| | CLES presurvey | | CLES postsurvey | | | | | |
| Question | Statesville (n=9) | Mooresville (n=19) | Statesville (n=9) | Mooresville (n=19) | pre% | post% | *z* | *p* |
| PR1 | 5 | 5 | 3 | 7 | 36 | 36 | 0 | 0.5 |
| CV3 | 5 | 3 | 8 | 12 | 29 | 71 | -3.2 | .0007 |
| PR7 | 3 | 2 | 5 | 7 | 18 | 43 | -2.03 | .021 |
| CV9 | 3 | 9 | 7 | 13 | 43 | 71 | -2.16 | .015 |
| PR13 | 6 | 7 | 6 | 12 | 46 | 64 | -1.34 | 0.09 |
| CV15 | 0 | 2 | 4 | 6 | 7 | 36 | -2.6 | .005 |
| PR19 | 3 | 3 | 5 | 11 | 21 | 57 | -2.71 | .0031 |
| CV21 | 1 | 2 | 5 | 14 | 11 | 68 | -4.38 | 0* |
| PR25 | 3 | 3 | 4 | 8 | 21 | 43 | -1.72 | .043 |
| CV27 | 4 | 8 | 9 | 15 | 43 | 86 | -3.35 | .0004 |
| CV32 | 6 | 8 | 7 | 16 | 50 | 82 | -2.54 | .006 |

*Note:* Test completed via TI-84 SE with *z* scores rounded to the hundredths
* value provide by TI-84 due to 9-digit float

Next, two population proportion *z*-tests were completed on each question comparing pre/post survey data for each of the control and treatment groups with success defined as responding Almost Always or Often (AAO) on positively worded questions. The test designed as a left-tailed test assuming an alternate hypothesis of %AAOpre < %AAOpost and α = .05. Data for the number AAO responses for each section, along with results of the test, can be found in Tables 12 and 13. Except for questions PR1 and PR13 in the control, all questions were found to be significant, $p < .05$. However, for the treatment groups, all questions showed a significant increase in PR and CV, $p < .005$.

Table 13
*Two Population Proportion Test for Treatment Groups – Positive Wording*

| | Number of AAO Reponses | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | CLES presurvey | | CLES postsurvey | | % of AAOs | | | |
| Question | Statesville (n=19) | Mooresville (n=23) | Statesville (n=19) | Mooresville (n=23) | pre % | post % | *z* | *p* |
| PR1 | 0 | 9 | 16 | 20 | 21 | 60 | -5.91 | 0* |
| CV3 | 6 | 10 | 14 | 14 | 38 | 57 | -2.62 | .0044 |
| PR7 | 2 | 4 | 16 | 20 | 14 | 48 | -6.55 | 0* |
| CV9 | 4 | 8 | 15 | 14 | 29 | 55 | -3.71 | .0004 |
| PR13 | 9 | 15 | 15 | 20 | 57 | 71 | -2.63 | .0043 |
| CV15 | 4 | 6 | 12 | 10 | 24 | 43 | -2.7 | .0035 |
| PR19 | 3 | 6 | 14 | 15 | 21 | 0.476 | -4.38 | 0* |
| CV21 | 6 | 5 | 11 | 13 | 26 | 38 | -2.88 | .002 |
| PR25 | 2 | 5 | 14 | 16 | 17 | 45 | -5.06 | 0* |
| CV27 | 13 | 15 | 18 | 21 | 67 | 79 | -2.99 | .0014 |
| CV32 | 10 | 12 | 17 | 22 | 52 | 69 | -4.16 | 0* |

Notes: Test completed via TI-84 SE, *z* scores rounded to the hundredths.
* value provide by TI-84 due to 9-digit float

Last, two population proportion *z*-tests were completed on each negatively worded question comparing pre/post survey data for each of the control and treatment groups with success defined as responding Seldom or Almost Never on negatively worded questions. The test designed as a left-tailed test assuming an alternate hypothesis of %SANpre < %SANpost and $\alpha = .05$. Data for the number SAN responses for each section, along with results of the test, can be found in Tables 14 and 15. All questions were found to be significant, $p < .05$.

Table 14
*Two Population Proportion Test for Control Groups – Negative Wording*

| | Number of SAN Reponses | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | CLES presurvey | | CLES postsurvey | | * of SANs | | | |
| Question | Statesville (n=9) | Mooresville (n=19) | Statesville (n=9) | Mooresville (n=19) | pre% | post% | *z* | *p* |
| PR30 | 1 | 3 | 6 | 7 | 29 | 82 | -4.03 | 0 |
| PR37 | 4 | 3 | 6 | 11 | 25 | 61 | -2.70 | .0035 |
| CV39 | 3 | 5 | 7 | 16 | 36 | 75 | -2.95 | .0016 |

Notes: Test completed via TI-84 SE, *z* scores rounded to the hundredths.
* value provide by TI-84 due to 9-digit float

Table 15
*Two Population Proportion Test for Treatment Groups – Negative Wording*

| | Number of Seldom and Almost Never (SAN) Reponses | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | CLES presurvey | | CLES postsurvey | | % of SANs | | | |
| Question | Statesville (n=19) | Mooresville (n=23) | Statesville (n=19) | Mooresville (n=23) | pre% | post% | *z* | *p* |
| PR30 | 4 | 7 | 14 | 20 | 26 | 81 | -5.03 | .0* |
| PR37 | 7 | 8 | 17 | 21 | 36 | 90 | -5.20 | 0* |
| CV39 | 10 | 9 | 17 | 18 | 45 | 83 | -3.64 | .0001 |

Notes: Test completed via TI-84 SE, *z* scores rounded to the hundredths.
* value provide by TI-84 due to 9-digit float

**Summary of Findings**

The use of a critical pedagogy impacted course success differently on the two campuses and indicated that there was no immediate measurable impact on overall persistence on either campus. The Statesville treatment did not show significant gains in course grades, final exam grades, or the CAOS posttest. While there was not a measurable increase in course success that does not imply that other academic-related gains were not achieved. Questions measuring critical voice and personal relevance from the CLES survey were used to measure MP. The critical voice scale measures if the climate of the classroom allows students to question the overall pedagogical plan and methods used, increasing accountability and student empowerment (Taylor et al., 1994). The personal relevance scale measures if the students are using their everyday experiences in a meaningful context to develop mathematical knowledge (Taylor et al., 1994). In Statesville, both areas used to measure MP showed a significant increase overall. Pfaff and Weinburg (2009) had a similar outcome when their active learning modules failed to increase students' understanding of statistical concepts but had a noticeable impact on student engagement resulting in positive feedback of the course projects.

The Mooresville treatment had very different results. The results show significant gains in course grades, final exam grades, and the CAOS posttest but had mixed results on the impact on MP. In Mooresville, the use of a critical pedagogy increased personal relevance but had a negative impact on critical voice. Again, this is consistent with other findings. Staples (2013) found her students at a small boarding school in Massachusetts benefitted greatly from a Calculus lesson about wealth distribution in the United States, both in engagement and mathematical understanding. However, while Staples' students were moved by the lesson and understood the mathematics, they still struggled to understand their own economic positions and the role they played in maintaining that social structure, despite the school's emphasis on community service and lessons that paid particular attention to race and class throughout the curriculum (Staples, 2013).

# Chapter Five: Conclusions

This chapter will analyze the results of the study through the lens of the conceptual framework and discuss how the findings relate to the literature. Following the analysis, gaps in and limitations of the study will be identified and addressed. Last, implications of the research will be explored, including recommendations for future research.

## Analysis of the Results

This NCGD study was designed to measure the impact of teaching for social justice in a community college statistics classroom on content mastery, course completion, and mathematical empowerment. To do so, two of the four courses included in the study were instructed using the Teaching Statistics for Social Justice framework (TSfSJ). The TSfSJ framework is a critical pedagogy that is justice orientated. Merging social constructivism, critical mathematics, and mathemacy, it teaches statistics in a way that requires students to identify and reflect on the injustices that surround them while mastering content. Specifically, TSfSJ aims to cultivate a critical awareness and an empowered agency to inspire future social action (Swalwell, 2013).

Two research questions guided the study: (1) does implementing a critical pedagogy increase student success? and (2) does use of a critical pedagogy increase the student's sense of mathematical empowerment? A variety of data was collected via instructor created projects, online homework sets, tests, and surveys to measure the impact of TSfSJ. Course grades, final exam grades, and pre/post CAOS tests were used to examine student success, with success defined as completing the course with a C or higher (a numerical score of 70 or higher). Mathematical Empowerment refers the student's ability to identify the need for and ability to apply statistics to real world scenarios and was measured using the CLES.

**Research Question One.** Does implementing a critical pedagogy increase student success? Will the use of TSfSJ increase courses grades and overall persistence? To measure student success data relating to the participants the final course grade and final exam grade were collected. Additionally, the CAOS was implemented as a pretest/posttest test to measure growth. Due to differences in demographics, separate control groups and treatment groups for each campus were used.

***Statesville Sections.*** The Statesville area is more rural and more economically challenged than the Mooresville area. Statesville is 72% White and 25% Black/African American, with median incomes of approximately $48,000 and $25,000, respectively. These differences appear to have contributed to very different results between the two groups. No statistically significant difference between course grades, final exam grades, the CAOS pretest/posttest scores, or overall persistence was found between the Statesville control and treatment group. This is not uncommon when using critical pedagogies with marginalized populations. Though the Statesville area is more diverse, the population of White residents are largely living below the national median income.

While Whites living below or near the poverty level fill the effect of growing income inequality and classism, they are often still unaware of the privilege granted by their whiteness. Whiteness refers to the taken-for-granted, socially developed system of privilege for those who are white. Many white people have no awareness of their whiteness, how it was constructed, or the role they play in sustaining the inequities it creates (Gillborn, 2005). This is often particularly true for whites living in poverty as, while they benefit from whiteness, they still feel forgotten. The wealth inequality between whites leaves those who are not members of the elite class on the fringes of the “master class”; they do not receive the same social benefits of those living in higher income households but are still not subjected to institutional racism and the majority of systemic unjust practices. For those left clinging to the edges of white privilege, “whiteness has not brought freedom and dignity to the majority of European Americans in this country…It does not exempt people from exploitation; it reconciles them to it. It is for those who have nothing else” (Ignatiev, 1997, pp. 199-200).

In turn, when challenged to address their whiteness, white students who do not feel the positive effects of class privilege often will resist engaging in a social justice framework. They may do this by not taking part in conversations or withdrawing from the class (Allen & Rossatto, 2009; Welton, Harris, La Londe, & Moyer, 2015; Swalwell, 2015). Allen and Rossatto (2009) note that white students in critical classrooms often do poorly on class assignments, both in terms of understanding the concepts and completing the assignments, and they may resist deeper readings of the materials. This appears to have impacted overall success in the course, as well as persistence.

Non-white students may also resist the use of critical pedagogies. This may be due to a lack of awareness of the inequality that surrounds them, leading them to believe they are not oppressed and to continue to participate in and support their own oppression (Freire, 1970/2005; Giroux, 1983). However, more common, due to the awareness of the oppressive structures, non-white students may develop a “contradiction of identity” (Allen & Rossatto, 2009; Welton et al., 2015). In order to gain privilege, those in oppressed groups adopt and internalize the culture of the oppressor. Many groups over time have ‘redefined’ themselves as white to join the privilege group (Allen & Rossatto, 2009; Gillborn, 2005; Ignatiev, 1997). To challenge ‘being white’ means to risk losing privilege, but ‘being white’ continues to reproduce the inequitable model, making fully embracing a critical pedagogy a very personal and painful process. “As long as they live in the duality in which *to be* is *to be like*, and *to be like* is *to be like the oppressor"* minority students will be unable to embrace a critical pedagogy (Freire, 1970/2005, p. 48).

***Mooresville Sections.*** While inequalities relating to race, gender, ethnicity, and sexual orientation exist, "disparities between the rich and poor are the most deeply embedded, durable trend in US civic life" (Swalwell, 2015, p. 492). Income and educational quality and attainment are undeniably linked. Those with higher levels of education have lower levels of unemployment and earn a higher yearly income making it easier to save and create wealth (Wolla & Sullivan, 2017). However, income serves as a catalyst for and as a result of education. School systems in lower socioeconomic areas often receive less funding than schools who serve students in nonpoverty areas (Chingos & Blagg, 2017). The lack of funding, coupled with the higher needs of students living in poverty, have a negative impact on academic outcomes, more so than their family environment (Aikens & Barbarin, 2008). Students from low socioeconomic areas enter high school at an average literacy skill that is five years behind high income students and achieve lower success rates in STEM courses (American Psychological Association, 2019). Due to these factors, they are more likely to dropout and less likely to have access to college information and opportunities to develop vocational skills.

Mooresville is roughly 90% White, 28115 is nearly 8% Black or African American, and 28117 is approximately 3% Black or African American. However, both zip codes are significantly more affluent than Statesville. Compared to Statesville, the median income for Whites in 28115 is 26% higher and for Black or African Americans it is 64% higher. The difference is even more stark in 28117, with Whites earning 65% more and Black or African Americans earning a staggering 114% more than those in Statesville. It is also important to note the discrepancy between the median incomes of Whites and Black/African Americans in

Mooresville is narrower. In Statesville, the median income of Whites is 91% higher than Black/ African Americans; it is 46% in 28115 and 25% in 28117.

Swalwell (2013) notes that affluent students who are educated with a critical pedagogy focused on social justice will learn to “mobilize their privilege on behalf of and act in alliance with marginalized people” instead of withdrawing from it or attempting to capitalize on it via service hours or a more moving college essay (p. 3). By focusing on awareness, agency, and action, higher income students can learn to move past *participatory citizenship*, helping those in need through charity, to a *justice-orientated citizenship* where they will actively engage in structural change to address social injustices (Swalwell, 2015). Given this information, it was not surprising that the wealth difference between the two areas had a noticeable impact on the results with the Mooresville treatment group showing a statistically significant difference from the control group in course grades, final exam grades, and the CAOS pretest/posttest, but no impact on persistence.

**Research Question Two.** Does use of a critical pedagogy increase the student’s sense of mathematical empowerment?  Mathematical Empowerment (MP) refers the student’s ability to identify the need for and ability to apply statistics to real world scenarios. The CLES was used to measure MP. The Constructivist Learning Environment Survey (CLES) is a 42-item questionnaire which uses a 5-point Likert scale measuring students' classroom experiences in six categories, but this study only examined two, Personal Relevance (PR) and Critical Voice (CV). Seven PR questions measured a students’ perceived relevance of the content learned to their out-of-school experiences and seven CV questions measured the overall classroom climate and the extent to which students felt a sense of empowerment (Taylor, 2013).

Pre-surveys asked students to rate prior mathematics classes and post surveys asked students to rate the control or treatment course. Overall, all positively worded questions showed a statistically significance difference between the percentage of pooled “Always” or “Almost Always” responses for the control groups and the treatment groups. Likewise, negatively worded questions showed a statistically significance difference between the percentage of pooled “Seldom” or “Almost Never” responses for the control groups and the treatment groups. These findings imply that the TSfSJ framework has a significant impact on mathematical empowerment.

However, the two campuses yielded different, and interesting, results when positively worded questions were examined by campus. For positively worded questions, two questions in the Statesville control were found to be statistically significant, both in CV. The CV questions were CV15 “It was OK to complain about activities that were confusing” and CV27 “I was free to express my opinion.” No other changes in PR or CV were found. However, in the Statesville treatment, all questions in both categories were found to be statistically significant. This is consistent with the literature as critical pedagogies frequently increases agency among marginalized students (Freire 1970/2005; Gutstein, 2003; Leonard et al., 2010; Welton et al., 2015).

Results for the Mooresville sections for positively worded questions, while very different from Statesville, also was consistent with the literature. The control group for Mooresville showed a statistically significant difference in all CV questions and in three of the five positively worded PR questions. The PR questions that were not significant were PR1 “I learned about the outside world” and PR13 “I learned how mathematics can be part of my out-of-school life.” The Mooresville treatment produced opposite results. All PR questions and four of the six positively worded CV questions were significant. The two CV questions that lost significance were CV3 “It was OK to ask the teacher ‘why do we have to learn this?’ and CV15 “It was OK to complain about activities that were confusing.” This consistent with Swalwell’s assertion that affluent Whites, both consciously and unconsciously, may resist participating in any discussion or activity that may threaten their own social status (2015). For the Mooresville treatment, this was a conscious hesitation given that all PR questions were found significant. While TSfSJ helped them to uncover injustices, resulting in raised awareness and increased empowerment, they may have been unsure of how to ‘unpack’ those feelings.

For negatively worded questions, the findings were almost always significant. The single negatively worded CV question, CV39 “I felt unable to complain about anything,” was significant for all four groups. PR37 “What I learned had nothing to do with the world outside of school” was significant in all sections expect the Statesville control and PR30 “What I learned had nothing to do with my out-of-school life” was significant in all sections expect the Mooresville control. While, it appears to contradict the other findings due to the wording, the *t*-test confirms that the data for the treatment does vary from the control. It is with the aid of the two-population *z*-tests that we can see the significance is positive, as it shows an increase in the number of students who disagree with the statement, due the reverse scoring.

**Examining Student Engagement**

Use of a critical pedagogy with an emphasis on social justice issues has shown to raise student engagement in both statistics and mathematics courses through reflective inquiry (Cheng et al., 2018; Leonard et al., 2015; Lesser, 2007; Voss & Rickards, 2016; Wright, 2016). While student engagement is correlated with numerous positive academic outcomes, Zepke (2015) argues that the mainstream definition of student engagement and student success are too narrow.

Relying solely on generic indictors measured by statistics in a singular setting, the traditional definition of student engagement ignores outcomes that may occur outside the classroom or may not be measurable by a Likert scale. Taking a more holistic view of four measures of engagement – agency, success, wellbeing, and social justice – expands the desirable outcomes of student engagement to include the development of a critical consciousness, increased democratic participation, engaging in positive relationships with others, and critical active citizenship (Zepke, 2015). While you may see evidence of the beginnings of these skills in the classroom, the true effects may take years to surface and may not result in increased course grades.

The restrictive nature of traditional student engagement measurement techniques and tools were illuminated by this study. By simply measuring traditional areas with statistical tools, the true difference in levels of engagement between the control and treatment courses were not as obvious, as most questions measured a significant difference in the control and treatment courses and students' past mathematics courses. Lurking variables, like using a constructivist approach in both the control and treatment, may have minimized the measurable impact of the treatment. This appears to be case due to course evaluations administered by MCC and the researcher's observations.

All students at MCC receive an end of course evaluation through IOTA360 for each course they are enrolled in. The instructor has no ability to disable the evaluations and cannot access the data until grades are submitted. On the IOTA360 evaluations, students from the Mooresville treatment specifically commented on the power of the course content and its relationship to the real world and their lives. Additionally, during classroom discussions the Mooresville treatment asked probing questions about the content, the data, the vetting process, and laws surrounding the topics being presented. These outcomes are representative of an enhanced learning agency as they show critical reflection, awareness of the world, and increased personal control (Zepke, 2015).

The Statesville treatment, while less vocal, would comment to the researcher about discussions they had with friends and family about the topics examined in class. The Statesville treatment, while it did not result in higher levels of content mastery, showed a significant increase in the understanding of the unjust structures and examples of discrimination studied in class. This not only points to an enhanced learning agency, but also to increased democratic participation and wellbeing. The Statesville treatment began to analyze their definition of fairness, question social 'givens,' and engaged in relationships centered on ways of knowing.

Most notably, both treatment groups began to learn for social justice. The coursework raised their awareness that inequities exist, how to examine and address them through active engagement, and the importance of communication through the change process. Though academic achievement is an important goal, TSfSJ arguably resulted in more important student success indicators as "students learn[ed] to critically reflect on their experiences, ask questions about wider society, take personal control over their learning and speak back to what they consider to be social injustice" (Zepke, 2015, p. 1318).

**Limitations**

All studies have limitations. In addition to the limitations presented in relation to the NCGD, hypothesis testing presents two challenges. First is that null hypothesis significant testing is highly sensitive to sample size. When samples are small even strong effects can be found to be nonsignificant, resulting in a Type II error (failing to reject a false null) (Levine, Weber, Hullet, Park, & Linsey, 2008). Additionally, non-parametric tests, such as the Mann-Whitney U, are more likely than parametric test (i.e. - *t*-tests) to result in a Type II error, regardless of sample size (Field & Hole, 2003). Second, even if a null hypothesis is rejected, that does not necessarily ensure that the findings support the other hypothesis. While both these limitations can somewhat be addressed and minimized, there are other more pressing non-quantitative limitations.

First and foremost, regardless of the level of commitment to social justice, one cannot avoid or completely detach from their privilege. There were times during classroom discussions that ‘teachable moments’ were lost in an attempt to balance tension and avoid alienation of the material as participants were not first introduced to the idea of privilege or whiteness. A second limitation was the standardization of the curriculum set by the North Carolina Community College System, required department resources, and the collection of student data to measure student learning outcomes for the college accreditation process. This limited the time and scope of the material presented in the classroom. Third, grade level and prior college experience of the participants was not considered. The impact of TSfSJ may have had a profoundly different effect on traditional college students as opposed to early college or dual enrolled students and students repeating the course may have had an advantage. Last is the impact of the researcher’s authority in the classroom. While participants reported they were comfortable questioning content and speaking up for their rights, the researcher and the classroom instructor was the same individual which ultimately impacts the balance of power and authority (Giroux, 1983; 2004).

**Revisiting the Conceptual Framework**

Teaching statistics for social justice (TSfSJ) was the conceptual framework that guided the study. Using a NCGD, a control and treatment was assigned on both the Mooresville and Statesville campuses. The control pedagogy was a constructivist approach that emphasized real world applications but did not implement social justice issues. Teaching materials consisted of both textbook provided resources and instructor created examples and labs. The treatment, TSfSJ, used statistics to explore issues of social justice relating to race, class, gender, and sexual orientation through instructor created classroom examples and student lab**s**. All materials were "immediate and meaningful" as they were authentic, factual, accurate, and relevant to the area that the participants lived in to ensure they would link the content to their daily lives (Okazaki, 2005, p. 181).

However, two key elements were missing from the framework: preparation and praxis. Participants were not provided with an introduction to issues relating to social justice, the idea of privilege, or the relationship between power and oppression. Not providing participants with a common language or base knowledge of the issues and their place within them limited their ability to examine and critically reflect on the material and engage in critical discourse (Brown, 2004; Hackman, 2005; Okazaki, 2005,Welton et al., 2015). More critically, though, is the lack of action.

"Reflection only becomes truly critical when it leads to some form of transformative social action" (Brown, 2004, p. 86). While participants were analyzing and reflecting on their world, this does not automatically mean they will be moved to act (Hackman, 2005; Giroux, 2004; Swalwell, 2013). Freire (1970/2005) referred to the connection between reflection and action as praxis. It is through praxis, the merger of reflection and action, that one fully develops a critical consciousness and begins to transform the world (Brown, 2004; Freire, 1970/2005; Nagada et al., 2003). Adjusting the TSfSJ framework to include these elements will greatly increase its impact.

## Implications and Recommendations for Future Research

As the highest-level mathematics course taken in high school is the best predictor of success in a college-level mathematics course, mathematics courses often act a barrier between students and graduation (Adelman, 2006; Gupta et al., 2006). Due to this fact and the NCCCS funding formula that specifically assesses student success in college-level mathematics, institutions have explored numerous strategies to improve retention, persistence, and success in mathematics courses. While the data did not show a statistical difference between the control and treatment groups for both campuses, overall, the study showed that the use of a critical pedagogy is a promising tool to help increase achievement. While persistence in the individual course did not seem to be impacted, this can directly impact retention and completion as "by the end of the second calendar year of enrollment, the gap in credit generation in college-level mathematics between those who eventually earned bachelor's degrees and those who didn't is 71 to 38 percent" (Adelman, 2006, p. xix).

Future research should examine the impact of a critical mathematics pedagogy in other mathematics classes, with an emphasis on non-Calculus based courses such as North Carolina's Quantitative Literacy course. Only about 8% of students at two-year colleges take Calculus where nearly 28% of students take a statistics or liberal arts mathematics course (Blair et al., 2015). While overall the percentage of students taking Calculus and Precalculus at community colleges continues to rise, the rate of dual enrolled students taking those courses has dropped by over 40%. As community colleges serve an increasing population of dual enrolled students, this trend will continue. Additionally, a new pathway design for remedial mathematics is scheduled to begin during the 2020 – 2021 academic year in North Carolina that provides students with an alternative pathway to college-level courses, heavily emphasizing the Quantitative Literacy and Statistic courses.

Critical action research (CAR) projects should also be implemented during future research. First developed by Kurt Lewin, action research is a systematic approach to researching real-world problems by following a spiraling set of steps: learning, planning, action, and evaluation of action (Glennon, Hinton, Callahan, & Fischer, 2013; Wilson, 2017). Lewin, believing there was “no action without research; no research without action" (Lewin as cited in Adelman, 1993), developed it specifically to serve as a reflective process to improve social issues and spur social action (Altrichter, Kemmis, McTaggart, & Zuber-Skerritt, 2002).

Critical action research, also referred to as emancipatory action research, emphasizes identifying and addressing injustices (Wilson, 2017). When implemented in a classroom setting, through collective action and reflection students master content, develop critical thinking and communication skills, and build agency, making it less likely that they will become complacent about the injustices uncovered. Since notable increases in mathematical empowerment, as measured by the second research question, were found without CAR, implementing it could possibly have a profound impact on a student’s agency. As mathematical empowerment is arguably more important to math educational research than traditional academic success, the second research question could have been the sole research question for the study. However, in a field that is still dominated by the believe that mathematics is neutral, measuring increases in student learning using traditional methods is a powerful way to persuade math, statistics, and QL educators of the validity of using a critical pedagogy.

## Conclusion

Though the design of the study had limitations, it provided two promising findings. First, the study showed that the use of a critical pedagogy in an undergraduate introductory statistics classroom has the possibility of increasing overall course success. While some findings were not statistically significant, they did show grades trended in an upward direction. The difference between the course grades of 68 and a 72 may not seem like a significant increase, but represents a full letter grade increase from a D to a C. As students need a C or better in a course to fulfill prerequisite requirements for other courses or to have the course transfer to another higher education institution, even small, non-statistically significant increases may result in large, personally significant gains for the student. By earning transferrable credits, the probability a student completes their degree increases. As “educational attainment is a means of social mobility” this will have an impact on their future lives and career (Trusty & Niles, 2003).

Second, the study showed gains in mathematical empowerment in the treatment courses. While the control found the material relevant, students were not able to “read and write” the world with statistics and identify how mathematics touched their own lives. By implementing a critical pedagogy, we move away from teaching for a low-level of functional mathematical literacy to a more meaningful learning experience that allows students “to examine the systems and institutions that are in place and to use mathematics to evaluate and critique these systems and institutions as well as develop individual and social agency” (Leonard et al., 2010, p. 262).

The purpose of higher education has long been to help create well-rounded, forward thinking citizens; people that can gather and analyze information, use that information to address problems, and help society as a whole. As Stanton (1987) stated “we need a citizenry with a broad understanding of the interdependencies of peoples, social institutions, and communities and an enhanced ability both to draw upon and further develop this knowledge as they confront and solve human problems" (p. 7). Traditionally, mathematics courses have left exploring social problems to the social sciences, allowing classes like Sociology and History to shoulder the burden of teaching students how to identify and fight injustice. By teaching statistics for social justice, students not only learn how to be better employees for the workplace, but also better people for all places.

## References


Adelman, C. (1993). Kurt Lewin and the origins of action research. *Educational Action Research*, *1*(1), 7-24.

Adelman, C. (2006). *The toolbox revisited: Paths to degree completion from high school through college*. US Department of Education.

Aikens, N. L., & Barbarin, O. (2008). Socioeconomic differences in reading trajectories: The contribution of family, neighborhood, and school contexts. *Journal of Educational Psychology, 100*(2), 23 –251. DOI: 10.1037/ 0022-0663.100.2.235.

Allen, R. L., & Rossatto, C. A. (2009). Does critical pedagogy work with privileged students?. *Teacher Education Quarterly, 36*(1), 163-180.

Altrichter, H., Kemmis, S., McTaggart, R., & Zuber-Skerritt, O. (2002). The concept of action research. *The Learning Organization*, *9*(3), 125-131.

Alvarez-Bell, R. M., Wirtz, D., & Bian, H. (2017). Identifying keys to success in innovative teaching: Student engagement and instructional practices as predictors of student learning in a course using a team-based learning approach. *Teaching & Learning Inquiry*, *5*(2), 128–146.

American Psychological Association. (2019). *Education and socioeconomic status* [Fact sheet]. Retrieved from https://www.apa.org/pi/ses/resources/publications/education.aspx.

Amineh, R. J., & Asl, H. D. (2015). Review of constructivism and social constructivism. *Journal of Social Sciences, Literature and Languages*, *1*(1), 9-16.

Andrade-Molina, M. (2017). Be the best version of yourself! OECD’s promises of welfare through school mathematics. In A. Cronaki (Ed.), *Proceedings of the Ninth International Mathematics Education and Society Conference*, 2, 393-400. Retrieved from https://www.mescommunity.info/mes9b.pdf.

Apple, M. W. (1992). Do the standards go far enough? Power, policy, and practice in mathematics education. *Journal for Research in Mathematics Education*, *23*(5), 412-431.

Attewell, P., Lavin, D., Domina, T., & Levey, T. (2006). New evidence on college remediation. *The Journal of Higher Education*, *77*(5), 886-924.

Bahr, P. R. (2010). Preparing the underprepared: An analysis of racial disparities in postsecondary mathematics remediation. *The Journal of Higher Education*, *81*(2), 209-237.

Bailey, T. (2012). Can community colleges achieve ambitious graduation goals? In A.P. Kelly & M. Schneider (Eds.), *Getting to graduation: The completion agenda in higher education* (2nd ed., pp. 73-101). Maryland: John Hopkins University Press.

Baldwin, C. (2017). *The completion agenda in community colleges: What it is, why it matters, and where it's going*. Maryland: Rowman & Littlefield.

Bartell, T. G. (2013). Learning to teach mathematics for social justice: Negotiating social justice and mathematical goals. *Journal for Research in Mathematics Education*, *44*(1), 129-163.

Bennett, C. A. (2014). Creating cultures of participation to promote mathematical discourse. *Middle School Journal*, *46*(2), 20-25.

Ben-Zvi, D., & Garfield, J. (2008). Introducing the emerging discipline of statistics education. *School Science and Mathematics*, *108*(8), 355-361.

Bettinger, E. P., Boatman, A., & Long, B. T. (2013). Student supports: Developmental education and other academic programs. *The Future of Children*, *23*(1), 93-115.

Blair, R., Kirkman E., & Maxwell, J. (2015). *Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States: Fall 2015 CMBS Survey*. Rhode Island: American Mathematical Society. Retrieved from https://www.ams.org/profession/data/cbms-survey/cbms2015-Report.pdf.

Bragg, D. D., & Durham, B. (2012). Perspectives on access and equity in the era of (community) college completion. *Community College Review*, *40*(2), 106-125.

Brelias, A. (2015). Mathematics for what? High school students reflect on mathematics as a tool for social inquiry. *Democracy and Education, 23*(1), 4.

Brock, T. (2010). Young adults and higher education: Barriers and breakthroughs to success. *The Future of Children*, *20*(1), 109-132.

Brown, K. M. (2004). Leadership for social justice and equity: Weaving a transformative framework and pedagogy. *Educational Administration Quarterly*, *40*(1), 77-108.

Brown, R. (2009). Teaching for social justice: Exploring the development of student agency through participation in the literacy practices of a mathematics classroom. *Journal of Mathematics Teacher Education, 12*(3), 171-185.

Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher, 18*(1), 32-42.

Burbules, N. C., & Berk, R. (1999). Critical thinking and critical pedagogy: Relations, differences, and limits. In T. Popkewitz & L. Fendler (Eds.), *Critical theories in education: Changing terrains of knowledge and politics* (pp. 45-65). New York: Routledge.

Campbell, D. T., & Stanley, J. C. (1963). *Experimental and quasi-experimental designs for* *research*. Hopewell, N.J.: Houghton Mifflin Company

Carver, R., Everson, M., Gabrosek, J., Horton, N., Lock, R., Mocko, M., Rossman, A., Roswell, G.H., Velleman, P., Witmer, J., & Wood, B. (2016). *Guidelines for assessment and instruction in statistics education (GAISE) college report 2016*. Retrieved from https://www.amstat.org/asa/files/pdfs/GAISE/GaiseCollege_Full.pdf.

Cheng, S., Ferris, M., & Perolio, J. (2018). An innovative classroom approach for developing critical thinkers in the introductory statistics course. *The American Statistician*, *72*(4), 354-358.

Chingos, M. M., & Blagg, K. (2017). *Do poor kids get their fair share of school funding?* Washington, DC: Urban Institute. Retrieved from https://www.urban.org/sites/default/files/publication/90586/school_funding_brief.pdf

Christensen, L. B., Johnson, B., & Turner, L. A. (2014). *Research methods, design, and analysis* (12th ed.). Upper Saddle River, NJ: Pearson.

Clotfelter, C. T., Ladd, H. F., Muschkin, C., & Vigdor, J. L. (2015). Developmental education in North Carolina community colleges. *Educational Evaluation and Policy Analysis, 37*(3), 354-375.

Cobb, G. W., & Moore, D. S. (1997). Mathematics, statistics, and teaching. *The American Mathematical Monthly*, *104*(9), 801-823.

Cokley, K., & Awad, G. H. (2013). In defense of quantitative methods: Using the "master's tools" to promote social justice. *Journal for Social Action in Counseling & Psychology*, *5*(2), 26-41.

Community College, N.C. Gen. Stat. § 115D-31.3 (2016). Retrieved May 14, 2019, from https://www.ncleg.net/enactedlegislation/statutes/html/bychapter/chapter

_115d.html

Constructivist Learning Environment Survey (CLES). (2017). Retrieved July 27, 2017, from http://stelar.edc.org/instruments/constructivist-learning-environment-survey-cles

Cook, T. D., Campbell, D. T., & Shadish, W. (2002). *Experimental and quasi-experimental designs for generalized causal inference*. Boston, MA: Houghton Mifflin.

Cutler, D. M., & Lleras-Muney, A. (2008). Education and health: Evaluating theories and Evidence. In J. House, R. Schoeni, G. Kaplan, & H. Pollack (Eds.), *Making Americans healthier: Social and economic policy as health policy* (pp. 29 -60). New York: Russell Sage Foundation.

Delmas, R., Garfield, J., Ooms, A., & Chance, B. (2007). Assessing students' conceptual understanding after a first course in statistics. *Statistics Education Research Journal, 6*(2), 28-58.

Dewey, J. (1938). *Experience and education*. New York: Touchstone.

Dewey, J. (2008). The supremacy of method. In J.A. Boydston (Series Ed.), *The later works of John Dewey*, *Volume 4, 1925-1953: 1929: The quest for certainty* (pp. 178-202). Carbondale, Illinois: Southern Illinois University Press. (Original work published in 1929).

Dewey, J. (2013). My pedagogic creed. In D. J. Flinders & S. J. Thornton (Eds.), *The curriculum studies reader* (4th ed., pp. 33-40). New York: Routledge. (Original work published in 1897).

Dimitrov, D. M., & Rumrill Jr, P. D. (2003). Pretest-posttest designs and measurement of change. *Work*, *20*(2), 159-165.

Dörfler W., & Mclone R.R. (1986). Mathematics as a school subject. In B. Christiansen, A.G.

Howson, & M. Otte (Eds.), *Perspectives on mathematics education* (pp. 49-97). Massachusetts: Kluwer Academic Publishers.

Dougherty, K. J., & Reddy, V. (Eds.). (2013). *ASHE higher education report special issue: Performance funding for higher education: What are the mechanisms? What are the impacts.* Retrieved from https://ebookcentral.proquest.com.

Erickson, A. W. (2016). Rethinking the numerate citizen: Quantitative literacy and public issues. *Numeracy*, *9*(2), 1-20. DOI: 10.5038/1936-4660.9.2.4.

Ernest, P. (2002). Empowerment in mathematics education. *Philosophy of Mathematics Education Journal*, *15*(1), 1-16.

Fassinger, R., & Morrow, S. L. (2013). Toward best practices in quantitative, qualitative, and mixed-method research: a social justice perspective. *Journal for Social Action in Counseling & Psychology*, *5*(2), 69-83.

Field, A., & Hole, G. (2003). *How to design and report experiments*. California: Sage.

Frankenstein, M. (1983). Critical mathematics education: An application of Paulo Freire's epistemology. *Journal of Education*, *165*, 315-339.

Frankenstein, M. (1990). Incorporating race, gender, and class issues into a critical mathematics literacy curriculum. *The Journal of Negro Education*, *59*(3), 336-347.

Frankenstein, M. (2013). Reading the world with math: Goals for a critical mathematical literacy curriculum. In E. Gutstein & B. Peterson (Eds.), *Rethinking mathematics: Teaching social justice by the numbers* (2nd ed., pp. 30-41). Wisconsin: Rethinking Schools, Ltd.

Frankenstein, M., & Powell, A. B. (1989). Empowering non-traditional college students: On social ideology and mathematics education. *Science and Nature (9/10),* 100-112.

Freire, P. (2005). *Pedagogy of the oppressed,* 30th anniversary edition (M. B. Ramos, Trans). New York: Continuum International Pub Group. (Original work published in 1970)

Garner, B. E., & Garner, L. E. (2001). Retention of concepts and skills in traditional and reformed applied calculus. *Mathematics Education Research Journal*, *13*(3), 165–84.

Glennon, C., Hinton, C., Callahan, T., & Fischer, K. W. (2013). School-Based Research. *Mind, Brain, and Education*, *7*(1), 30-34.

Gillborn, D. (2005). Education policy as an act of white supremacy: Whiteness, critical race theory and education reform. *Journal of Education Policy*, *20*(4), 485-505.

Giroux, H. A. (1983). *Theory and resistance in education: A pedagogy for the opposition*. Massachusetts: Bergin & Garvey.

Giroux, H. A. (2004). Public pedagogy and the politics of neo-liberalism: Making the political more pedagogical. *Policy Futures in Education*, *2*(3-4), 494-503.

Gonzalez, L. (2009). Teaching mathematics for social justice: Reflections on a community of practice for urban high school mathematics teachers. *Journal of Urban Mathematics Education*, *2*(1), 22-51.

Goldrick-Rab, S., & Cook, M. A. E. (2011). College students in changing contexts. In P.G. Altbach, R.O. Berdahl, & P.J. Gumport (Eds.) *American higher education in the twenty-first century: Social, political, and economic challenges* (2nd ed., pp. 254-278). Maryland: John Hopkins University Press.

Gordon, M. (2009). Toward a pragmatic discourse of constructivism: Reflections on lessons from practice. *Educational Studies, 45*(1), 39-58.

Gupta, S., Harris, D. E., Carrier, N. M., & Caron, P. (2006). Predictors of student success in entry-level undergraduate mathematics courses. *College Student Journal*, *40*(1), 97.

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education, 34*(1), 37–73. https://doi-org.proxy006.nclive.org/10.2307/30034699.

Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice*. New York: Taylor & Francis. Kindle edition.

Gutstein, E., & Peterson, B. (Eds.). (2013). *Rethinking mathematics: Teaching social justice by the numbers* (2nd ed.). Wisconsin: Rethinking Schools, Ltd.

Hackman, H. W. (2005). Five essential components for social justice education. *Equity & Excellence in Education, 38*(2), 103-109.

Hahs-Vaughn, D. L., Acquaye, H., Griffith, M. D., Jo, H., Matthews, K., & Acharya, P. (2017). Statistical literacy as a function of online versus hybrid course delivery format for an introductory graduate statistics course. *Journal of Statistics Education*, *25*(3), 112-121.

Haney, J. J., Wang, J., Keil, C., & Zoffel, J. (2007). Enhancing teachers' beliefs and practices through problem-based learning focused on pertinent issues of environmental health science. *The Journal of Environmental Education*, *38*(4), 25-33.

Harrison, L. (2015). Teaching social justice through mathematics: A self-study of bridging theory to practice. *Middle Grades Review, 1*(1), 5.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., ... Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher*, *25*(4), 12-21.

Horn, L., Nevill, S., & Griffith, J. (2006). *Profile of undergraduates in U.S. postsecondary education institutions--2003-04, with a special analysis of community college students. [electronic resource]: statistical analysis report.* United States Dept. of Education, National Center for Education Statistics. Retrieved from http://eric.ed.gov/?id=ED491908.

Hunt, S. D. (1990). Truth in marketing theory and research. *Journal of Marketing, 54*(3), 1-15.

Ignatiev, N. (1997). The new abolitionist. In W. Williams, C. West, & N. Ignatiev (Author), *I'm Ofay, You're Ofay* (Extract, pp. 199-203). *Transition*, (73), 199-203. DOI: 10.2307/2935453

Johnson, B., & McClure, R. (2004). Validity and reliability of a shortened, revised version of the Constructivist Learning Environment Survey (CLES). *Learning Environments Research*, *7*(1), 65-80.

Kaya, C., & Kaya, S. (2017). Prospective teachers' educational beliefs and their views about the principles of critical pedagogy. *Journal of Education and Learning*, *6*(4), 181-190.

Kelly, A. P., & Schneider, M. (Eds.). (2012). *Getting to graduation: The completion agenda in higher education*. Maryland: John Hopkins University Press.

Kenny, D. A. (1975). A quasi-experimental approach to assessing treatment effects in the nonequivalent control group design. *Psychological Bulletin, 82*(3), 345-362.

Kolesnikova, N. (July 2009). From community college to a bachelor's degree and beyond: how smooth is the road? *The Regional Economist*, 10-11. Retrieved from https://files. stlouisfed.org/files/htdocs/publications/regional/09/07/community_college.pdf

Kopko, E. M., & Crosta, P. M. (2016). Should community college students earn an associate degree before transferring to a 4-year institution? *Research in Higher Education*, *57*(2), 190-222.

Kuh, G. D., Cruce, T. M., Shoup, R., Kinzie, J., & Gonyea, R. M. (2008). Unmasking the effects of student engagement on first-year college grades and persistence. *The Journal of Higher Education*, *79*(5), 540-563.

Kwon, O. N., Rasmussen, C., & Allen, K. (2005). Students' retention of mathematical knowledge and skills in differential equations. *School Science & Mathematics, 105*(5), 227–239.

Ladson-Billings, G. (1995). Toward a theory of culturally relevant pedagogy. *American Educational Research Journal*, *32*(3), 465-491.

Leonard, J., Brooks, W., Barnes-Johnson, J., & Berry III, R. Q. (2010). The nuances and complexities of teaching mathematics for cultural relevance and social justice. *Journal of Teacher Education*, *61*(3), 261-270.

Leonard, J., & Moore, C. M. (2014). Learning to enact social justice pedagogy in mathematics classrooms. *Action in Teacher Education*, *36*(1), 76-95.

Lesser, L. M. (2007). Critical values and transforming data: Teaching statistics with social justice. *Journal of Statistics Education*, *15*(1), 1-21.

Lesser, L. M., & Blake, S. (2007). Mathematical power: Exploring critical pedagogy in mathematics and statistics. *Journal for Critical Education Policy Studies*, *5*(1). Retrieved from http://www.jceps.com/wp-content/uploads/PDFs/05-1-13.pdf.

Levine, T. R., Weber, R., Hullett, C., Park, H. S., & Lindsey, L. L. M. (2008). A critical assessment of null hypothesis significance testing in quantitative communication research. *Human Communication Research, 34*(2), 171-187.

Lew, L. Y. (2010). The use of constructivist teaching practices by four new secondary school science teachers: A comparison of new teachers and experienced constructivist teachers. *Science Educator*, *19*(2), 10-21.

Logue, A. W., Watanabe-Rose, M., & Douglas, D. (2016). Should students assessed as needing remedial mathematics take college-level quantitative courses instead? A randomized controlled trial. *Educational Evaluation and Policy Analysis*, *38*(3), 578-598

Maxwell, J. (1992). Understanding and validity in qualitative research. *Harvard Educational Review*, *62*(3), 279-301.

Morris, S. B. (2008). Estimating effect sizes from pretest-posttest-control group designs. *Organizational Research Methods*, *11*(2), 364-386.

Muijs, D. (2004). *Doing quantitative research in education with SPSS*. Thousand Oaks, CA: Sage.

Musu-Gillette, L., de Brey, C., McFarland, J., Hussar, W., Sonnenberg, W., & Wilkinson-Flicker, S. (2017). Status and trends in the education of racial and ethnic groups 2017 (NCES 2017-051). *National Center for Education Statistics*. Retrieved from https://files.eric.ed.gov/fulltext/ED574873.pdf.

Nagda, B. R. A., Gurin, P., & Lopez, G. E. (2003). Transformative pedagogy for democracy and social justice. *Race, Ethnicity and Education*, *6*(2), 165-191.

National Council of Teachers of Mathematics. (2000). *Principals and standards for school mathematics*. Retrieved from http://www.nctm.org/uploadedFiles/Standards_ and_Positions/PSSM_ExecutiveSummary.pdf.

National Center for Education Statistics. (2018). *The Condition of Education 2018* (NCES 2018-144), Characteristics of Postsecondary Faculty.

Nix, R. K., Fraser, B. J., & Ledbetter, C. E. (2005). Evaluating an integrated science learning environment using the Constructivist Learning Environment survey. *Learning Environments Research*, *8*(2), 109-133.

North Carolina Association for Developmental Education. (2017). Career and college ready graduates: *NC legislation* [PowerPoint slides]. Retrieved from ncade.net/wp-content/uploads/2018/04/RISEPD.pptx

North Carolina Community College System. (2017). State board of community colleges code. Retrieved from https://www.nccommunitycolleges.edu/sbcccode

North Carolina General Assembly. (2016). *Funding for North Carolina's community colleges: A description of the current formula and potential methods to improve efficiency and effectiveness* (Report Number 2013-09). Retrieved from http://www.ncleg.net/PED/Reports/ documents/CCFunding/CC_Report.pdf

OECD. (2014). *PISA 2012 Results (Volume IV): Students' Financial Literacy*. PISA. Paris: OECD Publishing. DOI: https://doi.org/10.1787/9789264270282-en.

Ogbuehi, P. I., & Fraser, B. J. (2007). Learning environment, attitudes and conceptual development associated with innovative strategies in middle-school mathematics. *Learning Environments Research*, *10*(2), 101-114.

Oxford University Press. (2019). Definition of social justice. Retrieved from https://en.oxforddictionaries.com/definition/social_justice.

Okazaki, T. (2005). Critical consciousness and critical language teaching. *Second Language Studies*, *23*(2), 174-202.

Pennington, H., & Milliron, M. D. (2010). *Completion by Design Concept Paper*. Seattle, WA: Bill & Melinda Gates Foundation. Retrieved from https://www.completionbydesign.org/s/article/Completion-by-Design-Concept-Paper-September-2010.

Pew Research Center. (2016). *On views of race and inequality, blacks and whites are worlds apart*. Retrieved from https://www.pewsocialtrends.org /2016/06/27/1-demographic-trends-and-economic-well-being/

Pierce, K. B., & Hernandez, V. M. (2015). Do mathematics and reading competencies integrated into career and technical education courses improve high school student state assessment scores? *Career and Technical Education Research*, *39*(3), 213–229.

Pfaff, T. J., & Weinberg, A. (2009). Do hands-on activities increase student understanding?: A case study. *Journal of Statistics Education*, *17*(3). DOI: 10.1080/10691898.2009.11889536

Punch, K. F. (2013). *Introduction to social research: Quantitative and qualitative approaches* (3rd ed). California: Sage.

Quarles, C. L., & Davis, M. (2017). Is learning in developmental math associated with community college outcomes?. *Community College Review*, *45*(1), 33-51.

Regents of University of Minnesota. (2006). *The CAOS test*. Retrieved from https://apps3.cehd.umn.edu/artist/caos.html

Reichardt, C. (2009). Quasi-experimental design. In R. E. Millsap & A. Maydeu-Olivares (Eds.), *The SAGE handbook of quantitative methods in psychology* (pp. 47-72). London: SAGE Publications Ltd. DOI: 10.4135/9780857020994.n3

Roksa, J. (2012). Equalizing credits and rewarding skills: Credit portability and bachelor's degree attainment. In A.P. Kelly & M. Schneider (Eds.), *Getting to graduation: The completion agenda in higher education* (2nd ed., pp. 201-222). Maryland: John Hopkins University Press.

Romberg, T. A. (1992). Further thoughts on the standards: A reaction to Apple. *Journal for Research in Mathematics Education, 23*(5), 432-437.

Rouncefield, M. (1995). The statistics of poverty and inequality. *Journal of Statistics Education*, *3*(2), DOI: 10.1080/10691898.1995.11910491.

Ruggs, E., & Hebl, M. (2012). Diversity, inclusion, and cultural awareness for classroom and outreach education.  In B. Bogue & E. Cady (Eds.), *Apply research to practice (ARP) resources*. Retrieved March 30, 2017 from https://www.engr.psu.edu/AWE/ARPAbstracts/DiversityInclusion/ARP_DiversityInclusionCulturalAwareness_InfoSheet.pdf

Rumsey, D. J. (2002). Statistical literacy as a goal for introductory statistics courses. *Journal of Statistics Education*, *10*(3), DOI: 10.1080/10691898.2002.11910678

Rutherford, A., & Rabovsky, T. (2014). Evaluating impacts of performance funding policies on student outcomes in higher education. *The ANNALS of the American Academy of Political and Social Science*, *655*(1), 185-208.

Sabbag, A. G. & Zieffler, A. (2015). Assessing learning outcomes: An analysis of the Goals-2 instrument. *Statistics Education Research Journal*, *14*(2), 93-116.

Schield, M. (2005, April). Statistical literacy: An evangelical calling for statistical educators. Paper presented at the 2005 International Statistical Institute conference, Sydney, Australia.

Schoenfeld, A. H. (2002). Making mathematics work for all children: Issues of standards, testing, and equity. *Educational Researcher*, *31*(1), 13-25.

School Report Card for Mooresville High School. (2019). Retrieved from https://ncreportcards.ondemand.sas.com/src/school?school=491312&year=2017&lng=en

School Report Card for Lake Norman High. (2019). Retrieved from https://ncreportcards.ondemand.sas.com/src/school?school=490335&year=2017&lng=en

School Report Card for Statesville High. (2019). Retrieved from https://ncreportcards.ondemand.sas.com/src/school?school=490354&year=2017&lng=en

Skovsmose, O. (1994). Towards a critical mathematics education. *Educational studies in mathematics, 27*(1), 35-57.

Skovsmose, O. (1998). Linking mathematics education and democracy: Citizenship, mathematical archaeology, mathemacy and deliberative interaction. *Zentralblatt für Didaktik der Mathematik*, *30*(6), 195-203.

Skovsmose, O. (2004). Critical mathematics education for the future. In S. Lerman (Ed.), *Encyclopedia of mathematics education*. Dordrecht: Springer. DOI: 10.1007/978-94-007-4978-8_34. Retrieved from https://www.researchgate.net/profile/Ole_Skovsmose/publication/252205485_Critical_Mathematics_Education_for_the_Future/links/573f17a608ae9ace84133a8c.pdf.

Smith, M. L. (2006). Overcoming theory-practice inconsistencies: Critical realism and information systems research. *Information and Organization*, *16*(3), 191-211.

Spell, S. (2016, August 19). Multiple Measures. Retrieved from https://www.nccommunitycolleges.edu/student-services/multiple-measures

Stanton, T. K. (1987, October 15). *Liberal arts, experiential learning and public service: Necessary ingredients for socially responsible undergraduate education*. Paper presented at the Annual Conference of the National Society for Internships and Experiential Education, Smugglers' Notch, Vermont.

Staples, M. (2013). Integrals and equity: A math lesson prompts new awareness for prep school students—and their teacher. In E. Gutstein & B. Peterson (Eds.), *Rethinking mathematics: Teaching social justice by the numbers* (pp. 181-188). Wisconsin: Rethinking Schools, Ltd.

Steen, L. A. (Ed.). (2001). *Mathematics and democracy: The case for quantitative literacy*. Princeton, NJ: Woodrow Wilson Foundation.

Steen, L. A. (2004). Everything I needed to know about averages, I learned in college. *Peer Review*, *6*(4), 4-8.

Steiner, P. M., Wroblewski, A., & Cook, T. D. (2009). Randomized experiments and quasi-experimental designs in educational research. In K.E Ryans & J.B. Cousins (Eds.), *The Sage handbook of educational evaluation* (pp. 75-95). Thousand Oaks, CA: Sage.

Stinson, D. W. (2004). Mathematics as "gate-keeper" (?): Three theoretical perspectives that aim toward empowering all children with a key to the gate. *Mathematics Educator, 14*(1), 8-18.

Stinson, D. W., Bidwell, C. R., & Powell, G. C. (2012). Critical pedagogy and teaching mathematics for social justice. *The International Journal of Critical Pedagogy*, *4*(1), 76-94.

Swalwell, K. (2013). “With great power comes great responsibility”: Privileged students’ conceptions of justice-oriented citizenship. *Democracy and Education*, *21*(1), 5.

Swalwell, K. (2015). Mind the civic empowerment gap: Economically elite students and critical civic education. *Curriculum Inquiry*, *45*(5), 491-512.

Taie, S., & Goldring, R. (2017). *Characteristics of public elementary and secondary school teachers in the United States: Results from the 2015-16 national teacher and principal survey first look (NCES 2017-072)*. U.S. Department of Education. Washington, DC: National Center for Education Statistics.

Talja, S., Tuominen, K., & Savolainen, R. (2005). " Isms" in information science: constructivism, collectivism and constructionism. *Journal of Documentation*, *61*(1), 79-101.

Taylor, M. T. (2013). Replacing the 'teacher-proof' curriculum with the 'curriculum-proof' teacher: Toward more effective interactions with mathematics textbooks. *Journal of Curriculum Studies*, 45*(3)*, 295-321. DOI:10.1080/00220272.2012.710253.

Taylor, P. C., & Fraser, B. J. (1991, April). *CLES: An instrument for assessing constructivist learning environments*. Paper presented at the annual meeting of the National Association for Research in Science Teaching, Lake Geneva, WI.

Taylor, P. C., Fraser, B. J., & Fisher, D. L. (1997). Monitoring constructivist classroom learning environments. *International journal of educational research, 27*(4), 293-302.

Taylor, P. C., Fraser, B.J, & White, L.R. (1994, April). *CLES: An instrument for monitoring the development of constructivist learning environments*. Paper presented at the annual meeting of the American Educational Research Associations, New Orleans, LA.

Tishkovskaya, S., & Lancaster, G. A. (2012). Statistical education in the 21st century: A review of challenges, teaching innovations and strategies for reform. *Journal of Statistics Education*, *20*(2), 1-56. DOI: 10.1080/10691898.2012.1188964.

Trochim, W.K. (2006). The Nonequivalent Groups Design [Online image]. Retrieved February 2, 2019 from http://www.socialresearchmethods.net/kb/quasnegd.php

Triola, M. F. (2015). *Essentials of statistics* (5th ed.). Massachusetts: Pearson Addison Wesley.

Tristan, J.M.B (2013). Interview: Henry Giroux. *Globe Education Magazine*, 20-23. Retrieved from http://www.globaleducationmagazine.com/Global-Education-Magazine-School-Day-of-Non-violence-and-Peace.pdf

Trusty, J., & Niles, S. G. (2003). High-School math courses and completion of the bachelor's degree. *Professional School Counseling*, 7*(2)*, 99-107.

Tutak, F.A, Bondy E, & Adams, T.L. (2011) Critical pedagogy for critical mathematics education. *International Journal of Mathematical Education in Science and Technology, 42*(1), 65-74.

U.S. Census Bureau. (2018). *American Fact Finder. U.S. Census Bureau, 2012-2016 American Community Survey 5-Year Estimates*. Retrieved from https://factfinder.census.gov/faces/tableservices/jsf/pages/productview.xhtml?pid=ACS_16_5YR_S1903&prodType=table

Ukpokodu, O. N. (2007). Preparing socially conscious teachers: A social justice-oriented teacher education. *Multicultural Education*, *15*(1), 8-15.

Ukpokodu, O. N. (2011). How do I teach mathematics in a culturally responsive way?: Identifying empowering teaching practices. *Multicultural Education*, *19*(3), 47-56.

Vennix, J., Den Brok, P., & Taconis, R. (2017). Perceptions of STEM-based outreach learning activities in secondary education. *Learning Environments Research, 20*(1), 21-46.

Von Glaserfeld, E. (1984). An introduction to radical constructivism. In P. Watzlawick (Ed.), *The invented reality: How do we know what we believe we know? Contributions to constructivism* (6th ed., pp. 17-40). New York: Norton.

Voss, R., & Rickards, T. (2016). Using social justice pedagogies to improve student numeracy in secondary school education. *Journal of Education and Practice*, *7*(15), 40-47.

Vrasidas, C. (2000). Constructivism versus objectivism: Implications for interaction, course design, and evaluation in distance education. *International Journal of Educational Telecommunications*, *6*(4), 339-362.

Vygotsky, L.S. (1986). *Thought and language*. Massachusetts: MIT Press. (original work published in 1934). Retrieved from http://s-f-walker.org.uk/pubsebooks/ pdfs/Vygotsky_Thought_and_Language.pdf

Walker, W., & Plata, M. (2000). Race/gender/age differences in college mathematics students. *Journal of Developmental Education*, *23*(3), 24-32. Retrieved from http://www.jstor.org.proxy006.nclive.org/stable/42775780

Walpole, M. (2003). Socioeconomic status and college: How SES affects college experiences and outcomes. *Review of Higher Education: Journal of the Association for the Study of Higher Education, 27*(1), 45-73. DOI: http://dx.doi.org/10.1353/rhe.2003.0044

Walters, G. (2012). It's not so easy: The completion agenda and the states. *Liberal Education, 98*(1), 34-39. Retrieved from https://www.aacu.org/publications-research/periodicals/its-not-so-easy-completion-agenda-and-states.

Weiland, T. (2017). Problematizing statistical literacy: An intersection of critical and statistical literacies. *Educational Studies in Mathematics*, *96*(1), 33-47.

Welton, A. D., Harris, T. O., La Londe, P. G., & Moyer, R. T. (2015). Social justice education in a diverse classroom: Examining high school discussions about race, power, and privilege. *Equity & Excellence in Education, 48*(4), 549-570.

Wilson, E. (Ed.). (2017). *School-based research: A guide for education students*. California: Sage.

Winter, D. (2007). Infusing mathematics with culture: Teaching technical subjects for social justice. *New Directions for Teaching and Learning*, *2007*(111), 97-106. https://doi-org.proxy006.nclive.org/10.1002/tl.291

Wolla, S. A., & Sullivan, J. (2017). Education, income, and wealth. *Page One Economics*. Retrieved from https://research.stlouisfed.org/publications/page1-econ/2017/01/03/education-income-and-wealth/

Wright, D. B. (2006). Comparing groups in a before–after design: When t test and ANCOVA produce different results. *British Journal of Educational Psychology*, *76*(3), 663-675.

Wright, P. (2016). Social Justice in the Mathematics Classroom. *London Review of Education*, *14*(2), 104–118.

Wright, P. (2017). Teaching mathematics for social justice: transforming classroom practice. *Mathematics Education and Life at Times of Crisis*, 999.

Yilmaz, K. (2013). Comparison of quantitative and qualitative research traditions: Epistemological, theoretical, and methodological differences. *European Journal of Education*, *48*(2), 311-325.

Zepke, N. (2015). Student engagement research: Thinking beyond the mainstream. *Higher Education Research & Development*, *34*(6), 1311-1323.

# Appendix A

The following documents are examples of the labs used in the control groups for both Statesville and Mooresville.

## Chapter 4 Lab A: Dice Roll

**Goal:** *The purpose of this activity is to discover how basic probability works. We will use two dice to illustrate these principles.*

**Hypothesis:** Make a prediction as to how the distribution of the sums will look (skewed, normal, etc).

1.) **Roll the dice 36 times and record the sum of each roll in the table provided below.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |

2.) **Based on your data, calculate the following:**
   a. Mean
   b. Median
   c. Mode

3.) **Complete the following frequency table for the data:**

| Sum of the dice | Number of times each sum occurs | Fraction of each sum (out of 36 rolls) |
|---|---|---|
| **2** | | |
| **3** | | |
| **4** | | |
| **5** | | |
| **6** | | |
| **7** | | |
| **8** | | |
| **9** | | |
| **10** | | |
| **11** | | |
| **12** | | |

4.) **Compare your outcomes to your prediction. Was your prediction correct? Why do you think this happened?**

5.) **What number occurred most often? Least often? Why?**

6.) **Now, let us look at the possible outcomes for rolling each possible sum from two to twelve. Fill in the chart below to analyze this. (A tree diagram will be helpful!)**

| Sum of the die | List of ways sum can occur | Number of ways sum can occur | Probability of sum occurring |
|---|---|---|---|
| 2 | (1,1) | 1 | 1/36 |
| 3 | (1,2) (2,1) | 2 | 2/36 |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| 11 | | | |
| 12 | | | |

7.) **If you were to create a bar graph of the theoretical probabilities above, how would the bar graph look (skewed, normal, etc)?**

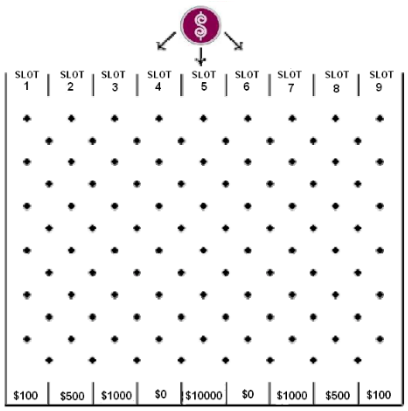
8.) **Which number should occur most often (theoretically)?**

9.) **How does your experiment earlier compare to the actual probabilities of rolling two dice? Did you have the same probabilities? Is your data close?**

10.) **What do you think would happen if we roll the dice 100 times? 1000 times?**

11.) **Other than gaming, what is another time that you would want to consider combinations and how often they may occur?** *(hint: a combination is when the order of how things happen doesn't matter. For example, 3+2 or 2+3 are both 5 so the order the pips appeared doesn't matter, just that they appeared.)*

**Chapter 5 Lab: Plinko**




Plinko is the most popular pricing game on *The Price is Right*. Debuting on January 3, 1983, it is played for a cash prize of up to $50,000 and also awards prizes valued under $100.
The contestant takes the chips they have earned up a set of stairs to the top of the Plinko board. The board is made up of a field of pegs, with each row offset from the previous row. Additionally, the sides have a zig-zag pattern. At the bottom of the board are nine slots marked symmetrically with the values $100, $500, $1,000, $0, $10,000.
One at a time, the contestant lays each Plinko chip flat against the top of the board and releases it. As the chip falls, it bounces off the pegs. The contestant wins whatever money corresponds with the slot the chip lands in, with a running total displayed on a scoreboard next to the Plinko board.
*Taken from http://priceisright.wikia.com/wiki/Plink*

## Part I: Slot 5

**Go to [http://www.kongregate.com/games/staplegun/plinko](http://www.kongregate.com/games/staplegun/plinko). If needed, login as [stats@mitchellccmail.com](mailto:stats@mitchellccmail.com) (password: stats). Select chip and place on the N to drop in slot 5.**

1. **Play the game 5 times. Record your results below.**

| Plinko Chip Drop | Amount you won |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |

**Total won: ______**

**2. Record the results of your five chips:**

| amount won | $0 | $100 | $500 | $1,000 | $10,000 |
|---|---|---|---|---|---|
| frequency | | | | | |

**3. Change these frequencies into probabilities. What is your expected value (also known as the mean)?**

| amount won | $0 | $100 | $500 | $1,000 | $10,000 |
|---|---|---|---|---|---|
| probability | | | | | |

**Expected value (Sum of $ times probability) =______**

## *Part II: Slot 4*

**What happens if you drop the chip from Slot 4 instead of Slot 5? Select chip and place it on the I for Slot 4.**

**4. Play the game 5 times assuming you dropped the chips from Slot 4. Use same methods of play from before.**

| Plinko Chip Drop | Amount you won |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |

**Total won: _____**

**5. Record the results of your five chips:**

| amount won | $0 | $100 | $500 | $1,000 | $10,000 |
|---|---|---|---|---|---|
| frequency | | | | | |

**6. Change these frequencies into probabilities. What is your expected value?**

| amount won | $0 | $100 | $500 | $1,000 | $10,000 |
|---|---|---|---|---|---|
| probability | | | | | |

**Expected value =** _____

## *Part III: Compare Slots*

**Below are the theoretical probabilities for Slot 5.**

**Table 1: Theoretical probabilities for Slot 5**

| x | $0 | $100 | $500 | $1,000 | $10,000 |
|---|---|---|---|---|---|
| P(x) | .39 | .03 | .11 | .24 | .23 |

**7. Use the theoretical probabilities to calculate the expected value and standard deviation for Slot 5.**

**Expected value =_____ Standard deviation = _____**

**8. How does your winnings for Slot 5 compare to the theoretical expected value? Are they similar?**

_____

**Below are theoretical probabilities for Slot 4 .**

### Table 2: Theoretical probabilities for Slot 4

| x | $0 | $100 | $500 | $1,000 | $10,000 |
|---|---|---|---|---|---|
| P(x) | .347 | .057 | .153 | .25 | .193 |

**9. Calculate the expected value and standard deviation using the theoretical probabilities for Slot**

**Expected value =______ Standard deviation =______**

**10. How do your winnings for Slot 4 compare to theoretical expected value?**

**_____**

**11. Which slot wins you more money in the long run: Slot 4 or Slot 5? How did you decide this?**

**_____**

**12. Which slot yields less variability (i.e.- is more predictable): Slot 4 or Slot 5? How did you decide this?**

**_____**

## *Part IIII: Simulations*

**You can simulate dropping a Plinko chip using a random number generator. A chip will bounce off 12 pegs before reaching the bottom of the board. The direction of the bounce will either be left or right. If we let left = 0 and right = 1, we can find out what our prize is using the sum of the simulation.**

**Go to [www.random.org/integers/](www.random.org/integers/) and generate 12 numbers between 0 and 1 in 2 columns. Add all the numbers. This is the number of right bounces. To find your location at the bottom of the Plinko, use the conversion table below (which accounts for reflections off the wall). The table assumes you drop the Plinko chip from Slot 5.**

### Table 3: Number of Right Bounces and Winnings

| Number of "right bounces" | winnings |
|---|---|
| 6 | $10,000 |
| 0,4,8,12 | $1,000 |
| 1,3,9,11 | $500 |
| 2,10 | $100 |

| | |
|---|---|
| 5,7 | $0 |

**13. Play the game 5 times. Record your results below.**

| Plinko chip Drop | Number of right bounces | Amount you won |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

**Total won: ______**

**14. What are the possible values for the number of right bounces with <u>this simulation</u>? ______**

**15. Are the possible values continuous or discrete? ______**

**16. As the chip can fall left or right, this is a binomial distribution. What are the values of *n* and *p* for this simulation, if *n* is the number of bounces and *p* is the probability it bounce to the right. ______**

**17. Run the Bionomialpdf with the n and p you identified in 18. Using the conversion table for Slot 5 (posted below) to construct the probability distribution for your winnings on one Plinko chip. (*hint: there are multiple ways to win $100. You win $100 if you get 2 right bounces OR 10 right bounces. Use probability rules!)***

**Table 4: Number of Right Bounces and Winnings**

| # of "right bounces" (y) | Winnings (x) |
|---|---|
| 6 | $10,000 |
| 0,4,8,12 | $1,000 |
| 1,3,9,11 | $500 |
| 2,10 | $100 |
| 5,7 | $0 |

***Bionimalpdf results (make sure to group outcomes!)***

| x | $0 | $100 | $500 | $1,000 | $10,000 |
|---|---|---|---|---|---|
| P(x) | | | | | |

**18. Compare your results to the theoretical probabilities for Slot 5. Are the same? Why or why not? *(hint: what does Bionomialpdf do? How did you get the probabilities?)***

**Chapter 6: Normal Probability Distributions ►Chapter 6 Lab Quiz: Denim Lengths**

Chapter 17
“There be a division of labor, of course.... A seamstress will make clothes. However, not all sizes can be made, so sizes will be based on the average height for females and males."
In the case of blue jeans, the following data was acquired. Assume the data is normally distributed. Round all answers to the hundredths.
Female Data in Inches

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 62 | 63.1 | 66.8 | 66.1 | 68.9 | 63.4 | 68.1 | 62 |
| 64.7 | 63.6 | 65.1 | 66.7 | 67.8 | 59.4 | 68.1 | 64.9 |
| 57 | 69.4 | 68 | 61.1 | 67.2 | 66.3 | 63.6 | 67.5 |
| 62.2 | 64.3 | 67.2 | 63.2 | 60.1 | 62.3 | 66.3 | 64.2 |

Question 1 What is the mean? Round to the hundredths.
Question 2 What is the Standard Deviation for the Sample? Round to the hundredths.
Question 3 What is the z score for a height of 72 inches? Round to the hundredths.
Question 4 What is the z score for a height of 60 inches? Round to the hundredths.
Question 5 What percent of female adults 5 feet (60 inches) or taller? Leave in decimal form rounded 4 decimal places (example- 82.56% would be .8256)
Question 6 What percent of female adults are 6 feet (72 inches) or taller?Leave in decimal form rounded 4 decimal places (example- 82.56% would be .8256)
Question 7 What percent of female adult heights are between 60 inches and 72 inches? Leave in decimal form and round 4 decimal places.
Question 8 What height, in inches, represents the "cut o " for the bottom 8%? Round to the all answers to the hundredths.
Question 9 What height, in inches, represents the cut off for the upper 6%?

Assume the data is normally distributed.
**Male Data in Inches**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 68.1 | 72.5 | 71.4 | 71.2 | 67.7 | 72 | 68 | 69 |
| 65.3 | 65.5 | 71.5 | 72.5 | 66 | 74.2 | 67.7 | 66.1 |
| 67.5 | 70.2 | 67.4 | 71.8 | 68.1 | 67.2 | 70.5 | 66 |
| 67.7 | 67 | 73.8 | 66.5 | 70.1 | 68.6 | 66.3 | 69.3 |

Question 10 What is the mean? Round to the hundredths.
Question 11 What is the Standard Deviation for the sample? Round to the hundredths.

Question 12 What is the z-score for 72 in height?
Question 13 What is the z score for the 60 in. height?
Question 14 What percent of male adults 5 feet (60 inches) or taller? Leave in decimal form rounded 4 decimal places (example- 82.56% would be .8256)
Question 15 What percent of male adults are 6 feet (72 inches) or taller? Leave in decimal form rounded 4 decimal places (example- 82.56% would be .8256)
Question 16 What percent of male adult heights are between 60 inches and 72 inches? Leave in decimal form rounded 4 decimal places (example- 82.56% would be .8256)
Question17 What is the height, in inches, that is cuto for the bottom 10% ? Round to the hundredths.

Question18 What is the height, in inches, that is cutoff for the upper 8%? Round to the hundredths

.

## Chapter 7 Lab B: Rivets

Boeing is teamed with Lockheed Martin, Pratt & Whitney and the U.S. Air Force to produce the F-22 Raptor air dominance fighter. The Raptor's unique combination of stealth, speed, agility, precision and situational awareness make it overwhelmingly effective in its combined air-to-air and air-to-ground mission capability. This aircraft is the next generation of precision strike aircraft replacing the F-117A.
Boeing has a contract with an outside company, Opus Enterprise Inc., to supply rivets. These rivets are used to fasten the wings to the fuselage (body) of the plane, etc. **The contract specifically requires the rivets have a mean diameter of 1.48 cm., 50% of the rivets must be usable, the standard deviation for all rivets should be .23 cm.**
~~**To ensure quality, you have an assistant examine 10 randomly selected rivets.**~~

1) As the rivet will work or not work, $\hat{p} = .5$. Assuming the data is normally distributed, construct a 95% confidence interval, using the sample of 10, to estimate the population proportion **p**. You may use the TI or formula card. Round to the hundredths. (Section 7.2)

2) Is it safe to assume that 50% of the rivets can be used? Why or why not?

3) The tolerance, **allowable error**, of useable rivets is .15. Using $\hat{p} = .5$ and the allowable error, find the sample size needed to construct a 95% confidence interval for the population proportion. (Section 7.2)

**As 10 is clearly not a sufficient sample size, you send your assistant to gather more data.**

4) Below are 90 diameters. Find a $\overline{x}$ and Sx. Round to the nearest hundredth.

***Hint: This is a table so you can copy and paste this into excel. There is a standard deviation formula : =STDEVS(cell1:celln).***

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1.189 | 1.364 | 1.104 | 1.325 | 1.502 | 0.923 | 2.216 | 2.309 | 0.657 | 0.929 |
| 2.142 | 1.375 | 1.631 | 1.949 | 1.481 | 1.076 | 0.743 | 2.076 | 1.977 | 1.416 |
| 1.34 | 1.83 | 1.466 | 1.711 | 2.214 | 1.736 | 1.815 | 1.273 | 1.661 | 1.595 |
| 1.135 | 1.399 | 1.814 | 1.374 | 1.34 | 0.911 | 1.259 | 1.198 | 0.289 | 1.911 |
| 0.813 | 1.472 | 1.484 | 0.346 | 0.843 | 1.036 | 1.793 | 2.284 | 1.589 | 0.983 |
| 1.825 | 1.387 | 1.632 | 2.125 | 1.297 | 1.113 | 1.405 | 1.805 | 1.975 | 0.569 |
| 1.893 | 1.557 | 1.22 | 1.209 | 1.646 | 1.814 | 1.74 | 1.307 | 0.977 | 1.344 |
| 2.003 | 1.832 | 1.343 | 0.591 | 2.195 | 2.007 | 0.966 | 1.655 | 1.68 | 1.268 |
| 0.998 | 2.48 | 1.938 | 1.119 | 1.218 | 1.662 | 1.372 | 0.854 | 0.858 | 0.995 |

$\overline{x}$ = Sx =

5) Assuming the **sample data** is essentially normal, using your **Sx and $\overline{x}$**, construct a 95% confidence interval to estimate the population mean **μ**. You may use the TI or formula card. Round to the hundredths. Use df = 90. (Section 7.3)

6) The contract has specified a population standard deviation of .23 cm. Using the sample mean and the given **population standard deviation of .23**, construct a 95% confidence interval to estimate **μ** assuming the data is normal You may use the TI or formula card. Round to the hundredths. (Section 7.3)

7) Using the n = 90 sample, find, with 95% confidence, the estimate of the population standard deviation. Use df = 90. (Section 7.4)

***** Remember:***

$$\frac{(n-1)s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_L^2}$$

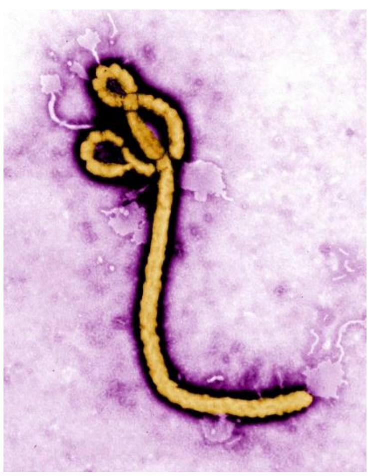
8) The contract asserts:
- 50% of the rivets must be usable
- The mean rivet diameter should be 1.48 cm

- The standard deviation for ALL rivets should be .23 cm

Based upon the contract's specifications for the rivets, does Boeing accept or reject the shipment of 200,000 rivets from Opus Enterprise, Inc.? Comment on whether the contract specifications were satisfied by the confidence intervals. **No math = no points!**

9) What is another scenario that you can use confidence intervals to verify/confirm/explore data sets?

## Chapter 8 Lab B: Ebola



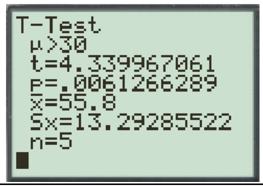
Eleven healthy people are abducted and unknowingly become part of the Ebola medical study. Five people are exposed to the virus and given Vaccine A, three people are exposed to the virus and injected with Vaccine B, and three people are not exposed to the virus or given any vaccine. The last group serves as a control group to test whether the virus can be spread through physical contact (sneezing, coughing, kissing, etc.)

The number of days for the patients to display symptoms related to the Ebola-Shiva virus are listed below.

| | |
|---|---|
| Vaccine A: | 37, 49, 57, 70, 66 |
| Vaccine B: | No subjects displayed any symptoms |
| Control Group: | 61, 39, 55 |

1) Test the following claims about the mean and standard deviation of the Vaccine A group. Use a 0.05 level of significance.
   a. It will take the Vaccine A group **more than 30 days** to display symptoms of the Ebola-Shiva virus.

| |
|---|
| **Step 1: Write hypothesis**<br>**$H_0$:** $\mu$ = 30 days<br>**$H_1$:** $\mu$ > 30 days ***Claim*** |
| **Step 2: Gather data and identify type of test.**<br><br>x-bar = 55.8<br>Sx = 13.3<br>The level of significance is 0.05. It is a right tail test. |
| **Step 3: Find the P-Value and test statistic using TI.** |




**Step 4: Test hypothesis**

Reject the Null as the p-value is less than the .05 significance level.
There is sufficient evidence to support the claim that symptoms will be displayed in more than 30 days.

b. The Ebola-Shiva virus will have a standard deviation of **7 day**s to display symptoms.

**Step 1: Write hypothesis**

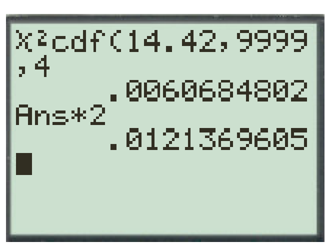
**$H_0$: $\sigma$ =7 days *Claim***
**$H_1$: $\sigma \neq$ 7 days**

**Step 2: Gather data and identify type of test and critical values.**

Sx = 13.29
It is a two tail test with df = 4
The level of significance is 0.05…. 0.975 to the right, .025 to the right.
Critical values are 0.484 and 11.143

**Step 3: Find $\chi^2$ and P-Value**

$$\chi^2 = \frac{4(13.29)^2}{7^2} \approx 14.42$$




**** For two tails, you must double P-value.****

| |
|---|
| **Step 4: Test hypothesis and state conclusion**<br><br>Reject the Null. The $\chi^2$ test statistic does is not between the critical values and alpha is less than .05. There is sufficient evidence to warrant the rejection of the claim the Ebola-Shiva virus will have a standard deviation of **7 day**s to display symptoms. |

***Using the first two questions as examples, complete the following four questions using the data provided.***

*"In the clean room, all ten were sprayed in the face…then half were given injections in the arm…and in four to six weeks, after the end of the Sydney Olympics, the plague would erupt worldwide".*

Ten subjects, all homeless, are exposed to the Ebola-Shiva virus. Five are not treated with any type of antibiotics. Five are treated with the antibiotics to treat a viral infection.

The number of days for the patients to display symptoms related to the Ebola-Shiva virus are listed below.

| | |
|---|---|
| Untreated: | 37, 42, 31, 29, 49 |
| Treated: | 49, 55, 79, 62, 68 |

2) Test the following claims about the mean and standard deviation of the **untreated group**. Use a 0.05 level of significance.

a. It will **take 30 days** to display symptoms of the Ebola-Shiva virus.

| |
|---|
| **Step 1: Write hypothesis**<br>**$H_0$:** $\mu$<br>**$H_1$:** $\mu$ |
| **Step 2: Gather data and identify type of test.** |
| **Step 3: Find the P-value and test statistic using TI.** |
| **Step 4: Test hypothesis and state conclusion** |

b. The Ebola-Shiva virus will have a standard deviation **of less than 7 days** to display symptoms.

| |
|---|
| **Step 1: Write hypothesis**<br><br>**$H_0$:** $\sigma$<br><br>**$H_1$:** $\sigma$ |
| **Step 2: Gather data and identify type of test and critical value(s).** |
| **Step 3: Find $\chi^2$ and P-Value.** |
| **Step 4: Test hypothesis and state conclusion** |

3) Test the following claims about the mean and standard deviation of the **treated group**. Use a 0.05 level of significance.

   a. It will take **more than 30 days** to display symptoms of the Ebola-Shiva virus.

| |
|---|
| **Step 1: Write hypothesis**<br>**$H_0$:** $\mu$<br>**$H_1$:** $\mu$ |
| **Step 2: Gather data and identify type of test.** |
| **Step 3: Find the P-Value and the test statistic using the TI.** |
| **Step 4: Test hypothesis and state conclusion** |

   b. The Ebola-Shiva virus will have a standard deviation of **more than** 7 days to display symptoms. Use a 0.05 significance level.

| |
|---|
| **Step 1: Write hypothesis**<br>**$H_o$:** $\sigma$<br>**$H_1$:** $\sigma$ |
| **Step 2: Gather data and identify type of test and critical value(s).** |
| **Step 3: Find $\chi^2$ and P-Value** |
| **Step 4: Test hypothesis and state conclusion** |

# Appendix B

The following documents are examples of the labs used in the treatment groups for both Statesville and Mooresville.

## Chapter 4 Lab A: Racial Profilin and Dice

Part I: Racial Profiling (?)
Nationally, including North Carolina, there have been numerous studies claiming that a disproportionate number of Black/African Americans are stopped in traffic stops. In 1999, North Carolina became the first state in the country to mandate the collection of data whenever a police officer stops a motorist (N.C.G.S. § 143B-903). Below is the data collected from January 2005 to November 2017 for Iredell County:

**Table 5: Initial Purpose of Traffic Stop by Driver's Sex, Race, and Ethnicity**

| **Purpose** | **White** | **Black** | **Native American** | **Asian** | Other | Total |
|---|---|---|---|---|---|---|
| Checkpoint | 103 | 19 | 0 | 1 | 12 | 135 |
| Driving While Impaired | 92 | 37 | 0 | 1 | 22 | 152 |
| Investigation | 846 | 255 | 4 | 8 | 102 | 1215 |
| Other Motor Vehicle Violation | 1860 | 566 | 3 | 58 | 179 | 2666 |
| Safe Movement Violation | 3826 | 1859 | 56 | 308 | 1331 | 7380 |
| Seat Belt Violation | 665 | 233 | 0 | 11 | 76 | 985 |
| Speed Limit Violation | 4135 | 1610 | 31 | 197 | 643 | 6616 |
| Stop Light/Sign Violation | 553 | 87 | 1 | 10 | 57 | 708 |
| Vehicle Equipment Violation | 3415 | 1365 | 15 | 89 | 541 | 5425 |
| Vehicle Regulatory Violation | 2643 | 734 | 1 | 23 | 190 | 3591 |
| Total - Both Genders by Race | 18138 | 6765 | 111 | 706 | 3153 | 28873 |
| Percent of Stops by Race | | | | | | 100% |

1. **Complete the bottom row of the table, to the nearest tenth of a percent, identifying what percentage of stops by race.**

2. **Below is the most recent U.S. Census Demographic Report for Iredell County. How does the demographic data compare to the traffic stop data? What is similar/different? What could account for these differences (if anything)?**

**Table 6:U.S. Census Demographic Report for Iredell County**

| Race | Percent |
|---|---|
| White | 82.7% |
| Black/African America | 12.2% |
| Native American/Alaska Native | 0.6% |
| Asian | 2.6% |
| Other (Native Hawaiian/Pacific Island/Two or More Races) | 1.9% |

Part II: Sum of Two Dice

**In this section, we will explore the sample space, probability, and distribution of the sum of two die (for later use, of course!). First, make a prediction about the distribution of the sums of the pips (normal, uniform, skewed):**

3. **"Roll" the dice 36 times and record the sum of each roll in the table provided below.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |

4. **Based on your data, calculate the following:**
   a. Mean
   b. Median
   c. Mode

5. **Complete the following frequency table for the data:**

| Sum of the dice | Number of times each sum occurs | Fraction of each sum (number over 36) |
|---|---|---|
| **2** | | |
| **3** | | |
| **4** | | |
| **5** | | |
| **6** | | |
| **7** | | |
| **8** | | |
| **9** | | |
| **10** | | |
| **11** | | |
| **12** | | |

6. **Compare your outcomes to your prediction. Was your prediction correct? Why do you think this happened?**

7. **What number occurred most often? Least often? Why do you think that is?**

**8. Now, let us look at the possible outcomes for rolling each possible sum from two to twelve. Fill in the chart below to analyze this. (A tree diagram will be helpful!)**

| Sum of the die | List of ways sum can occur | Number of ways sum can occur | Probability of sum occurring |
|---|---|---|---|
| 2 | (1,1) | 1 | 1/36 |
| 3 | (1,2) (2,1) | 2 | 2/36 |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| 11 | | | |
| 12 | | | |

**9. If you were to create a bar graph of the theoretical probabilities above, how would the bar graph look (skewed, normal, etc)?**

**10. Which number should occur most often (theoretically)?**

**11. How does your experiment earlier compare to the actual probabilities of rolling two dice? Did you have the same probabilities? Is your data close?**

**12. What do you think would happen if we roll the dice 100 times? 1000 times?**

Part III: Modeling using Dice

**We can use the findings from Part II to run a simulation for Part I. By using sums of the die to represent race (for example, let a sum of 2 represent and Asian driver), we can use the random dice roller to model traffic stops.**

13. **There are 36 sums that are possible. Assume that there are 36 people living in Iredell County. How many people, given the U.S. Census Demographics would be each race? Round to the nearest whole number.**

*(Due to small percentages, I needed to combine Other with Native America/Alaska Native)*

| Race | Percent | Of 36 People |
|---|---|---|
| White (W) | 82.7% | |
| Black/African America (B) | 12.2% | |
| Asian (A) | 2.6% | |
| Native American/Alaska Native/Other (O) | 2.5% | *.025*36 = .9 = 1 person* |

14. **As both Asian and Other categories are only represented by 1 person, I have preassigned them to a sum of 2 and sum of a 12. Given that Whites are represented by 30 people and Black/African American drivers are represented by 4, assign sums to represent White (W) and Black/African American (B) drivers.**

| **Sum of the Pips** | **# of ways Sum Occurs** | **Race Sum Represents** |
|---|---|---|
| **2** | 1 | **A** |
| **3** | 2 | |
| **4** | 3 | |
| **5** | 4 | |
| **6** | 5 | |
| **7** | 6 | |
| **8** | 5 | |
| **9** | 4 | |
| **10** | 3 | |
| **11** | 2 | |
| **12** | 1 | **O** |

**15. Using your experiment from Part I (data organized in Question 5), what percentage (to the nearest tenth of a percent) of times did your traffic stops result in stopping:**

   a. White drivers:
   b. Black/African America drivers:
   c. Asian drivers:
   d. Drivers from Other Races:

**16. How does your experiment compare to the Iredell County Sheriff's data and the U.S. Census Demographics?**

Part IV: Conclusions

**There's been a significant amount of research on this topic. UNC-Chapel Hill professor Frank Baumgartner gathered, analyzed, and published a large set of data on the topic, included a study. It's also been in the local news and a commission has been created.**

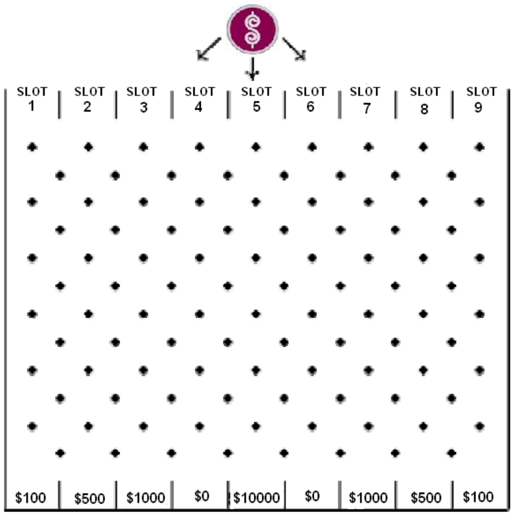
**17. Did your experiment imply there is reason for concern about racial profiling? Why or why not?**

**18. How can you improve upon the experiment? Do you think you would find a different result?**

***19.* What are some factors behind that real-life data that could impact it?** *(For example, times and locations…maybe)*

**20. Other than gaming and racial profiling, what is another time that you would want to consider combinations and how often they may occur?** *(hint: a combination is when the order of how things happen doesn't matter. For example, 3+2 or 2+3 are both 5 so the order the pips appeared doesn't matter, just that they appeared.)*

## Chapter 5 Lab: Plinko and Jury Duty




Plinko is the most popular pricing game on *The Price is Right*. Debuting on January 3, 1983, it is played for a cash prize of up to $50,000 and awards prizes valued over $100.
The contestant takes the chips they have earned up a set of stairs to the top of the Plinko board. The board is made of a field of pegs, with each row offset from the previous row. Additionally, the sides have a zigzag pattern. At the bottom of the board are nine slots marked symmetrically with the values $100, $500, $1,000, $0, $10,000.
One at a time, the contestant lays each Plinko chip flat against the top of the board and releases it. As the chip falls, it bounces off the pegs. The contestant wins whatever money corresponds with the slot the chip lands in, with a running total displayed on a scoreboard next to the Plinko board.

*Taken from http://priceisright.wikia.com/wiki/Plinko*

### Part I: Slot 5

**Go to [http://www.kongregate.com/games/staplegun/plinko](http://www.kongregate.com/games/staplegun/plinko). Select chip and place on the N to drop in slot 5. If needed, login as [stats@mitchellccmail.com](mailto:stats@mitchellccmail.com) (password: stats).**

1. **Play the game 5 times. Record your results below.**

| Plinko Chip Drop | Amount you won |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |

**Total won: _____**

**2. Record the results of your five chips:**

| amount won | $0 | $100 | $500 | $1,000 | $10,000 |
|---|---|---|---|---|---|
| frequency | | | | | |

**3. Change these frequencies into probabilities. What is your expected value (also known as the mean)?**

| amount won | $0 | $100 | $500 | $1,000 | $10,000 |
|---|---|---|---|---|---|
| probability | | | | | |

**Expected value (Sum of $ times probability) =_____**

4. **Below are the theoretical probabilities for Slot 5. Use the theoretical probabilities to calculate the expected value and standard deviation for Slot 5.**

**Table 7: Theoretical probabilities for Slot 5**

| x | $0 | $100 | $500 | $1,000 | $10,000 |
|---|---|---|---|---|---|
| P(x) | .39 | .03 | .11 | .24 | .23 |

**_Expected value =_____ Standard deviation = _____**

## *Part II: Slot 4*

**What happens if you drop the chip from Slot 4 instead of Slot 5? Select chip and place it on the I for Slot 4.**

13. **Play the game 5 times assuming you dropped the chips from Slot 4. Use same methods of play from before.**

| Plinko Chip Drop | Amount you won |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |

**Total won:** _____

**14. Record the results of your five chips:**

| amount won | $0 | $100 | $500 | $1,000 | $10,000 |
|---|---|---|---|---|---|
| frequency | | | | | |

**15. Change these frequencies into probabilities. What is your expected value?**

| amount won | $0 | $100 | $500 | $1,000 | $10,000 |
|---|---|---|---|---|---|
| probability | | | | | |

**Expected value =** _____

**16. Below are theoretical probabilities for Slot 4 Calculate the expected value and standard deviation**
**using the theoretical probabilities for Slot 4.**

**Table 8: Theoretical probabilities for Slot 4**

| x | $0 | $100 | $500 | $1,000 | $10,000 |
|---|---|---|---|---|---|
| P(x) | .347 | .057 | .153 | .25 | .193 |

**Expected value =_____ Standard deviation =_____**

## *Part III: Compare Slots*

**17. Theoretically, which slot wins you more money in the long run: Slot 4 or Slot 5? How did you decide this?**

**18. Theoretically, which slot yields less variability (i.e.- is more predictable): Slot 4 or Slot 5? How did you decide this?**

## *Part IV: Defining the Binomial Distribution*

**If you count the total number of right bounces of the pegs, you can predict where the Plinko chip will fall. For Slot 5, the number of right bounces needed for each payout slot is listed below:**

**Table 9: Number of Right Bounces and Winnings**

| Number of "right bounces" | Amount Won |
|---|---|
| 6 | \$10,000 |
| 0,4,8,12 | \$1,000 |
| 1,3,9,11 | \$500 |
| 2,10 | \$100 |
| 5,7 | \$0 |

**11. What are the possible values for the number of right bounces with <u>this simulation</u>?**

**12. Are the possible values continuous or discrete?**

**13. The chip can fall left or right making this is a binomial distribution. What are the values of *n* and *p* for this simulation, if *n* is the number of bounces and *p* is the probability it will bounce to the right.**

**14. Run the Binompdf with the n and p you identified in 13. Use the conversion table for Slot 5 (posted above) to construct the probability distribution table. (*hint: there are multiple ways to win \$100. You win \$100 if you get 2 right bounces OR 10 right bounces. Use probability rules!)***

***Bionimalpdf results (make sure to group outcomes!)***

| x | \$0 | \$100 | \$500 | \$1,000 | \$10,000 |
|---|---|---|---|---|---|
| P(x) | | | | | |

**15. Compare your results to the theoretical probabilities for Slot 5. Are the same? Why or why not? *(hint: what does Binompdf do? How did you get the probabilities?)***

## *Part V: Jury Selection and Binomial Distribution*

**Although juries are not selected solely by chance, comparing the actual jury to the composition of juries that would occur if jurors were selected at random can tell lawyers whether there are grounds to investigate the fairness of the jury selection process. The larger pool and panel are, supposedly, selected at random.**

**Figure 1: How Jury Selection Works**


**An historic case concerning jury selection, *Avery v. Georgia*, was brought to the U.S. Supreme Court in 1953. Originally, a jury in Fulton County, Georgia had convicted Avery, an African-American, of a serious felony. At the time of first trial, there were 165,814 African-Americans in the Fulton County population of 691,797 (making roughly 24% of the county was African-American).**

**A pool of 21,624 potential jurors (referred to as a jury pool) was generated. Roughly, 5.2% of the jury pool, 1,115 persons, were African-American. A jury panel of 60 people was selected from the jury pool. The jury panel, from which the 12 jurors are selected to serve on the jury, contained no African-Americans. Therefore, no African-Americans were on the jury for the *Avery v. Georgia* case.**

**16.** **Confirm that selection for the jury panel is a binomial distribution:**

- **a) Fixed number of trials? How many? _____**
- **b) Independent probability of success? What is p?_____**
- **c) Two outcomes? What are they?_____**

**17.** **What is the expected number (aka expected value) of African-Americans to be selected for the jury panel?**

**18.** **If 60 people for the jury panel were selected, at random, from the jury pool (where 5.2% are African- American), complete the binomial probability distribution table for the number of African-Americans (AA) on the jury panel. Round 4 decimal places.**

| Number of AA jurors | P(x) | Unusual? (P(x) ≤ .05) |
|---|---|---|
| 0 | | |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | .0008 | Yes |

| | | |
|---|---|---|
| **11** | **.0002** | **Yes** |
| **12 to 60** | **0** | **Yes** |

**19.** **Using binomcdf, what is the probability that the jury panel would consist of less than 12 AA jurors? What is the probability that is it 12 or more?**

**20.** **Assuming that the expected value of AA persons (rounded to the nearest whole person, mathematically) were selected for the jury panel, the AA would represent 5% of the jury panel. Given a probability of success of 5%:**

- **a) What is the probability no AA are selected for the 12-person jury?______**
- **b) What is the probability that at least one AA person would be selected for the 12-person jury? _____**

**21.** **While nearly 24% of the county's population was African-American, only 5.2% of the potential jurors were. The U.S. Supreme Court overturned Avery's conviction. Specifically, Justice Frankfurt wrote:**
*"The stark resulting phenomenon here was that somehow or other, despite the fact that over 5% of the slips [were African-American], no Negro got onto the panel of 60 jurors from which Avery's jury was selected. The mind of justice, not merely its eyes, would have to be blind to attribute such an occurrence to mere fortuity."*
**Describe the statistical evidence that you think might have been used by Avery's lawyers. _____**

Chapter 6: Normal Probability Distributions ►[Chapter 6 Lab Quiz - Part I: SAT Scores and Studen...]

NY –The Empire State

| | |
|---|---|
| Question1 | In New York, the mean SAT score was 1052 with a stdev of 188. If a student in NY scored a 1060, what would their z-score be? |
| Question2 | What percentage of students in NY scored higher than a 1060? Write as a decimal rounded 4 decimal places. |
| Question3 | A student who received a fee waiver in NY scored an average of 973. If a student in NY scored a 973, what would their z-score be? |
| Question4 | What percentage of students in NY scored lower than a 973? Write as a decimal rounded 4 decimal places. |
| Question5 | The school received the highest percentage of score reports was SUNY Stonybrook which is very competitive. For an “above average” of being selected you need to score in the Top 5% of NY test takers. What score do you need to be competitive for SUNY Stony Brook, the nearest whole point? |

NC – The Old North State

| | |
|---|---|
| Question6 | In NC, the mean SAT score was 1081 with a stdev of 181. If a student in NC scored a 1060, what would their z-score be? |
| Question7 | What percentage of students in NC scored less than a 1060? Write as a decimal rounded 4 decimal places. |
| Question8 | A student who did NOT received a fee waiver in NC scored an average of 1126. If a student in NC scored a 1126, what would their z-score be? |
| Qestion9 | What percentage of students in NC scored between 1060 and 1126? Write as decimal rounded 4 decimal places. |
| Question10 | UNC-Chapel Hill is the oldest (and often ranked the best) public college in the U.S. and highly competitive. For an “above average” of being selected you need to score in the Top 2% of NC test takers. What score do you need to be competitive for UNC-Chapel Hill to the nearest whole point? |

UT – The Beehive State

Question11 In UT, the mean SAT score was 1238 with a stdev of 185. If a student in UT scored a 1060, what would their z-score be?

Question12 What percentage of students in UT scored lower than a 1060? Write as a decimal rounded 4 decimal places.

Question13 A student who received a fee waiver in UT scored an average of 1171. If a student in UT scored a 1171, what would their z-score be?

Question14 What percentage of students in UT scored higher than an 1171? Write as decimal rounded 4 decimal places.

Question15 The University of Utah was school that received the most score reports. The university is lightly selective and students who score in the 25th percentile of UT SAT test takers are often accepted. What score do you need to be met this requirement to the nearest whole point?

MS – The Magnolia State

Question16 In MS, the mean SAT score was 1242 with a stdev of 187. If a student in MS scored a 1060, what would their z-score be?

Question17 What percentage of students in MS scored higher than a 1060? Write as decimal rounded 4 decimal places.

Question18 A student who did NOT received a fee waiver in MS scored an average of 1262. If a student in MS scored a 1262, what would their z-score be?

Question19 What percentage of students in MS scored between 1060 and 1262? Write as decimal rounded to 4 decimal places.

Question20 The Mississippi State University was school that received the most score reports. The university is lightly selective and students who score in the 70th percentile of MS SAT test takers are rated “above average” for acceptance. What score do you need to be met this requirement to the nearest whole point?

## Chapter 7 Lab B: Food Deserts

Food deserts are geographic areas where residents' access to affordable, healthy food options (especially **fresh** fruits and vegetables) is restricted or nonexistent due to the absence of grocery stores within convenient travelling distance. According to a report prepared for Congress by the Economic Research Service of the US Department of Agriculture in 2009, about 2.2% of all US households live more than one mile away from a supermarket and do not own a car.
In urban areas, access to public transportation may help residents overcome difficulties posed by distance, but economic forces have driven grocery stores out of many cities, making them so few and far between that an individual's food shopping trip may require taking several buses or trains. People often rely on convenience stores that normally only carry processed foods. In suburban and rural areas, public transportation is either very limited or unavailable, with supermarkets often many miles away from people's homes restricting access to any food items. Since the report, measurements have become more detailed and layered since access to food is impacted by poverty and geographic location. In addition to the 'usual' (counties, zip codes) researcher look at census tracts and consider multiple distances, with low-income tracts of particular interest. Low-income census tracts are where a significant number (at least 500 people) or share (at least 33 percent) of the population is greater than 1.0 mile from the nearest supermarket, supercenter, or large grocery store for an urban area or greater than 20 miles for a rural area. Under this measure in 2016, "an estimated 17.3 million people, or 5.6 percent of the U.S. population, live in low-income and low access tracts and are more than 1 mile or 20 miles from a supermarket."

---

**In this lab will we explore the level of food insecurity in North Carolina.**

1) In 2012, Iredell County had a food insecurity rate of 15%. There are 43 census tracts in Iredell County. Using a $\hat{p} = .15$, construct a 90% confidence interval, using the sample of 43, to estimate the population proportion **p**. Round three decimal places. (Section 7.2)

2) In North Carolina, the food insecurity rate was roughly 19% in 2012. Did Iredell County differ statistically from the state?

3) The error in the interval in question 1 was nearly 9%. When working with percentages and population it is important to have a small error (the population of NC in 2012 was 9.8 million). Using $\hat{p} = .15$ and an allowable error of .02, find the sample size needed to construct a 90% confidence interval for the population proportion. (Section 7.2)

**Census tracts are small sections within a county with long codes. North Carolina has over 2000 census tracts. As it is unreasonable for us to work with such a large data set, we will continue to explore food insecurity at the county and zip code levels. There are 100 counties and 508 zip codes in NC.**

4) Below is a random sampling of the number of supermarkets in 90 zip codes in 2012 (using all establishments). Find $\bar{x}$ and Sx. Round to the nearest tenth.

***Hint: This is a table so you can copy and paste this into excel. There is a standard deviation formula : =STDEVS(cell1:celln).***

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 5 | 4 | 3 | 1 | 1 | 5 | 2 | 1 | 3 |
| 3 | 3 | 6 | 8 | 6 | 1 | 3 | 4 | 8 | 5 |
| 9 | 1 | 3 | 3 | 2 | 1 | 1 | 3 | 1 | 2 |
| 5 | 1 | 3 | 1 | 3 | 2 | 1 | 1 | 4 | 1 |
| 1 | 1 | 1 | 5 | 5 | 6 | 1 | 1 | 1 | 1 |
| 1 | 2 | 3 | 1 | 3 | 1 | 6 | 4 | 22 | 1 |
| 3 | 8 | 4 | 1 | 13 | 8 | 7 | 1 | 1 | 4 |
| 1 | 3 | 1 | 4 | 4 | 1 | 1 | 2 | 2 | 5 |
| 1 | 4 | 1 | 2 | 9 | 7 | 1 | 4 | 3 | 2 |

$\bar{x}$ = Sx =

5) Since the data consists of n > 30, using your **Sx and $\bar{x}$**, construct a 95% confidence interval to estimate the population mean **μ**. Round to the nearest tenth. (Section 7.3)

6) Thanks to Excel, we know the standard deviation of all 508 zip codes. Using the sample mean and the **population standard deviation of 3.3**, construct a 95% confidence interval to estimate **μ**. Round to the tenth. (Section 7.3)

7) There were over 85,000 people in 2012 in NC's 100 counties that were impacted by food insecurity. Assuming that NC counties are a representative sample of all US counties, find, with 99% confidence, the estimate of the population standard deviation given that Sx = 24,357. You will need to use df = 100 as we do not have a df = 99, however n = 100. (Section 7.4)

$$\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$$

8) Explore the poverty map at the bottom of this page. Compare that to the interactive food insecurity map at this page. You can explore overlaps and various distances/combinations at this page. All maps are for the year 2015. What do you notice? What is the same/different? What other 'things' do you wonder about when you reflect on these maps? What conclusion do you draw from these maps?

9) What is another scenario that you can use confidence intervals to verify/confirm/explore data sets?

## Chapter 8 Lab B: Sentencing Times

**In this lab we will explore sentencing data gathered from the United States Sentencing Commission and Federal Justice Statistics about the mean sentence lengths and standard deviation in months for particular crimes for different demographics. Links to the data and referenced studies are included. Use the calculator for testing, when possible.**

1. In the 2014 fiscal year (FY), nationwide the mean incarceration sentence length for drug offenses was 77.6 months (table 5.4). However, Black/African Americans served a mean sentence length of 92.6 months. Assuming a standard deviation of 11 months (figure 2) from a study of 1112 cases, test the claim that B/AA serve more than national mean of 77.6 months using $\alpha = .05$.

| |
|---|
| **Step 1: Write hypothesis**<br>**$H_0$: μ**<br><br>**$H_1$: μ** |
| **Step 2: Identify type of test by tails and if it is z, t, or chi-square test.** |
| **Step 3: Find the P-value or critical value(s) and test statistic.** |
| **Step 4: State decision and conclusion** |

2. In FY2014, nationwide the mean incarceration sentence length for drug offenses was 77.6 months (table 5.4). However, males served a mean sentence length of 81.3 months. Assuming a standard deviation of 11 months (figure 2) due a study of 1112 cases, test the claim that males serve sentences that are equal to the national mean of 77.6 months using $\alpha = .05$.

| |
|---|
| **Step 1: Write hypothesis**<br>**$H_0$: μ**<br><br>**$H_1$: μ** |
| **Step 2: Identify type of test by tails and if it is z, t, or chi-square test.** |
| **Step 3: Find the P-value or critical value(s) and test statistic.** |
| **Step 4: State decision and conclusion** |

3. In FY2010, the nationwide mean incarceration sentence for weapon offenses was 84 months (table 5.4). However, women served a mean sentence of 50.7 months. The standard deviation for **all** cases was 38 months (pg. 248). Approximately 352 women received weapons offense sentences in FY2010. Test the claim that women serve a sentence that is less than the national mean of 84 months using $\alpha = .05$.

| |
|---|
| **Step 1: Write hypothesis**<br>**$H_0$: $\mu$**<br><br>**$H_1$: $\mu$** |
| **Step 2: Identify type of test by tails and if it is z, t, or chi-square test.** |
| **Step 3: Find the P-value or critical value(s) and test statistic.** |
| **Step 4: State decision and conclusion** |

**"To deal with sentencing disparity, the USSC created a Guideline system that required judges to use a sentencing table and to sentence defendants within a range of possible sentences based on the offense type and level and the defendants' criminal history… The Sentencing Guideline system was left virtually intact, until major reforms were instituted in 2003 and again in 2004/2005. These legal changes dramatically affected the amount of discretion that judges could exercise (p. 36).**

**You'll need to look up the critical values here: https://www.medcalc.org/manual/chi-square-table.php**

4. The study explored the mean sentence lengths and standard deviation of sentences of cases that followed guidelines and for those who did not. In 967 cases, guidelines were followed. Those cases had a standard deviation of 3.92 months (p. 40). Assuming the population standard deviation for all cases of the same nature was 11 months, test the claim that the standard deviation for the 967 guideline cases is less than the population standard deviation of 11 months with $\alpha = .025$ (use df = 950).

| |
|---|
| **Step 1: Write hypothesis**<br>$H_0$: $\sigma$<br><br>$H_1$: $\sigma$ |
| **Step 2: Identify type of test by tails and if it is z, t, or chi-square test.** |
| **Step 3: Find the P-value or critical value(s) and test statistic.** |
| **Step 4: State decision and conclusion** |

5. In 145 cases, guidelines were not followed, and the cases had a standard deviation of 16.36 months (same reference as previous question). Assuming the population standard deviation for all cases of the same nature was 11 months, test the claim the standard deviation for the 145 non-guideline cases is the same as the population standard deviation of 11 months with $\alpha = .05$.

| |
|---|
| **Step 1: Write hypothesis**<br>$H_0$: $\sigma$<br><br>$H_1$: $\sigma$ |
| **Step 2: Identify type of test by tails and if it is z, t, or chi-square test.** |
| **Step 3: Find the P-value or critical value(s) and test statistic.** |
| **Step 4: State decision and conclusion** |

## Appendix C

# CLES for Mathematics – Presurvey

**Using the ranking scale provided, rate your experience in your previous math courses.**
**(A) Almost Always (B) Often (C) Sometimes (D) Seldom (E) Almost Never**

*In my previous math courses…*

1. I learned about the world outside of school.

2. I learned that mathematics cannot provide perfect answers.

3. It was OK to ask the teacher "why do we have to learn this?"

4. I helped the teacher to plan what I'm going to learn.

5. I got the chance to talk to other students.

6. I looked forward to the learning activities.

7. New learning started with problems about the world outside of school.

8. I learned how mathematics has changed over time.

9. I felt free to question the way I'm being taught.

10. I helped the teacher decide how well my learning is going.

11. I talked with other students about how to solve problems.

12. The activities were among the most interesting at the school.

13. I learned how mathematics can be part of my out-of-school life.

14. I learned how the rules of mathematics were invented.

15. It was OK to complain about activities that are confusing.

16. I had a say in deciding the rules for classroom discussion.

17. I tried to make sense of other students' ideas.

18. The activities made me interested in mathematics.

19. I got a better understanding of the world outside of school.

20. I learned about the different mathematics used by people in other cultures.

21. It was OK to complain about anything that stops me from learning.

22. I had a say in deciding how much time I spend on an activity.

23. I asked other students to explain their ideas.

24. I enjoyed the learning activities.

25 . I learned interesting things about the world outside of school.

26 . I learned that mathematics is just <u>one</u> of many ways of understanding the world.

27 . I was free to express my opinion.

28. Other students asked me to explain my ideas.

29. I felt confused.

30. What I learn had nothing to do with my out-of-school life.

31. I learned that today's mathematics is different from the mathematics of long ago.

32. It was OK to speak up for your rights.

33. I had a say in deciding what would be on the test.

34. Other students explained their ideas to me.

<u>35</u>. The learning activities were a waste of time.

36. I had a say in deciding what activities I did.

37. What I learned had nothing to do with the world outside of school.

38. I learned that mathematics is about inventing rules.

39. I felt unable to complain about anything.

40. I had a say in deciding <u>how</u> my learning was assessed.

41. Other students paid attention to my ideas.

42. I felt tense.

# CLES for Mathematics – Post Survey

**Using the ranking scale provided, rate your experience in this course.**
**(A) Almost Always (B) Often (C) Sometimes (D) Seldom (E) Almost Never**

*In this class . . .*

1. I learned about the world outside of school.

2. I learned that mathematics cannot provide perfect answers.

3. It was OK to ask the teacher "why do we have to learn this?"

4. I helped the teacher to plan what I'm going to learn.

5. I got the chance to talk to other students.

6 I looked forward to the learning activities.

7 New learning started with problems about the world outside of school.

8 I learned how mathematics has changed over time.

9. I felt free to question the way I'm being taught.

10. I helped the teacher decide how well my learning is going.

11. I talked with other students about how to solve problems.

12. The activities were among the most interesting at the school.

13. I learned how mathematics can be part of my out-of-school life.

14. I learned how the rules of mathematics were invented.

15. It was OK to complain about activities that are confusing.

16. I had a say in deciding the rules for classroom discussion.

17. I tried to make sense of other students' ideas.

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32. It was OK to speak up for your rights.

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41. Other students paid attention to my ideas.

42. I felt tense.

Table 16

*CLES Questions*

| Scale | Questions |
|---|---|
| Personal Relevance | 1, 7, 13, 19, 25, 30*, 37* |
| Mathematical Uncertainty | 2, 8, 14, 20, 26, 31, 38 |
| Critical Voice | 3, 9, 15, 21, 27, 32, 39* |
| Shared Control | 4, 10, 16, 22, 28, 33, 40 |
| Student Negotiation | 5, 11, 17, 23, 29*, 34, 41 |
| Attitude | 6, 12, 18, 24, 29, 35*, 42* |

Note: *reverse scoring

## Vita

Toni Marie DiMella was born in Newark, Ohio and grew up in Flushing, New York. During her junior year of high school, she moved to Loch Sheldrake, New York where she would later graduate from SUNY Sullivan in December 1998 with an Applied Associate degree in Accounting. After transferring to Mount Saint Mary College (MSMC) from Manhattanville College, she was awarded the Bachelor of Arts degree in Mathematics in December 2000. In September 2001, she returned to MSMC to begin studying toward a Master of Science degree in Education to earn her license to teach Mathematics for grades 7 to 12. The M.S.Ed in Secondary Education was awarded in December 2003.

After teaching middle school for two years in New York, Toni relocated to North Carolina in July 2005 after accepting a position with Iredell-Statesville Schools. While teaching at the Collaborative College for Technology and Leadership Early College high school, she accepted an adjunct position with Mitchell Community College to teach Developmental Mathematics. In the August 2009, she would accept a full-time position with Mitchell to teach Development Mathematics while attending UNC-Charlotte to earn an additional 18 graduate level credits in Mathematics. During this time period, Toni also taught Credit Recovery Algebra II and created course content for North Carolina Virtual Public Schools.

The completion of the additional mathematics credits allowed Toni to transition from Developmental Mathematics to Curriculum Mathematics in August 2011. In the fall of 2013, she was accepted to Appalachian State University where she would complete an Educational Specialist degree in University and Community College Leadership in 2015 and an Educational Doctoral degree in Educational Leadership in 2019.

Toni is an animal lover, a runner, and enjoys trips to the beach. She hopes to one day open a sanctuary for senior cats and dogs. Toni resides in Charlotte, North Carolina with her husband, three rescue dogs, and a very large orange cat.